Hodge decomposition and Maximal Regularity for the Hodge Laplacian on homogeneous Besov spaces on $\mathbb{R}^n_+$

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Abstract

In 2020, Danchin, Hieber, Mucha and Tolksdorf introduced in **[arXiv:2011.07918]** a new point of view of homogeneous interpolation theory and the Da Prato-Grisvard Theorem to deal with L^q-maximal regularity with values in real interpolation spaces allowing global-in-time estimates for $q = 1, +\infty$. We will present here a combination of their theory with an appropriate construction of fractional homogeneous Sobolev spaces of differential forms on $\mathbb{R}^n_+$. This will lead, up to some compatibility conditions, to the Hodge (generalized Helmholtz) decomposition of homogeneous Besov spaces of differential forms $\dot{\mathrm{B}}^s_{p,q}(\mathbb{R}^n_+, \Lambda)$ provided $p \in (1, +\infty)$, $q \in [1, +\infty]$, $s \in (-1 + 1/p, 2 + 1/p)$, $n \geqslant 2$ is any dimension. As a consequence, we give some maximal regularity results for the generalized Hodge-Stokes, and Hodge-Maxwell systems on $\mathbb{R}^n_+$ such as a $\mathrm{L}^1_t(\dot{\mathrm{B}}^s_{p,1})$ one. As an application, we treat the global-in-time well-posedness of Hall-MHD system on $\mathbb{R}^n_+$ with minimal compatibility conditions on the boundary, and small datas, via the $\mathrm{L}^1_t(\dot{\mathrm{B}}^{n/p-1}_{p,1})$-maximal regularity, in arbitrary dimension $n \geqslant 2$, provided $p \in (n-1, 2n)$. The treatment of the linear theory allows an approach similar to the one performed by Danchin and Tan in **[arXiv:1911.03246]**.

If we have enough time, we will discuss several possible extensions.