

CIRM 2022

Abstract of the mini-course of Anne Pichon

Title. Lipschitz geometry of singularities

Abstract

It is well known that a real analytic germ $(X, 0) \subset (\mathbb{R}^N, 0)$ is topologically conical, i.e., homeomorphic as an embedded variety to the real cone over its link $X^{(\epsilon)} = X \cap S_\epsilon^{n-1}$. Now, a natural question is to study the metric evolution of the links $X^{(\epsilon)}$ when ϵ converges to zero : how do various regions shrink to zero ? A natural problem is then to build classifications of the germs up to local bi-Lipschitz homeomorphism, and what we call Lipschitz geometry of a singular space germ is its equivalence class in this category.

There are different approaches for this problem depending on the metric one considers on the germ. A germ $(X, 0) \subset (\mathbb{R}^N, 0)$ has actually two natural metrics induced from \mathbb{R}^n with its standard euclidean metric: the outer metric is defined by the restriction of the euclidean distance, while the inner metric is defined by the infimum of lengths of paths in V .

After a general introduction on Lipschitz geometry of singular germs, the course will focus on several recent results and open questions in the area. In particular, I will present a technics called *bubble trick* which is used in many recent works to explore Lipschitz invariants, and I will give a panorama of recent results on singular germs which are Lipschitz normally embedded, i.e., which have the same outer and inner Lipschitz geometries.

The course will be illustrated by many examples.