Coupler curves of moving graphs and counting realizations of rigid graphs

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A *calligraph* is a graph that for almost all edge length assignments moves with one degree of freedom in the plane, if we fix an edge and consider the vertices as revolute joints. The trajectory of a distinguished vertex of the calligraph is called its coupler curve. Each calligraph corresponds to an algebraic series of real curves in $\mathbb{P}^2$. I will present a description for the class of those curves in the Néron-Severi lattice of $\mathbb{P}^2$ blown-up at six complex conjugate points.

A graph is said to be *minimally rigid* if, up to rotations and translations, admits finitely many, but at least two, realizations into the plane for almost all edge length assignments. A minimally rigid graph can be expressed as a union of two calligraphs, and the number of its realizations is equal to the product of classes of those two calligraphs. I will show how one can apply those observations to produce an improved algorithm that counts the numbers of realizations. This, in turn, allows one to characterize invariants of coupler curves.

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