

FROM THE GEOMETRY OF INVARIANT DISTRIBUTIONS TO THE RIGIDITY OF PARTIALLY HYPERBOLIC DIFFEOMORPHISMS

MARTIN MION-MOUTON

ABSTRACT

To study rigidity phenomena, it is natural and has proved to be useful to consider geometric structures invariant by the dynamical system one wishes to study. Far to be rare, such structures appear naturally for differentiable dynamics in the form of invariant distributions of dynamical origin. Indeed, Oseledets theorem tells us that any volume-preserving diffeomorphism preserves some almost-everywhere defined *Lyapunov distributions*. These distributions are *a priori* only measurable, and several works seem to suggest that in lots of instances, the lack of regularity of these distributions is in fact the main obstacle to rigidity.

Concerning *contact-Anosov flows*, successive works of Ghys [Ghy87] (in dimension three) and Benoist-Foulon-Labourie [BFL92] (in any dimension) have for instance proved that the smoothness of the stable and unstable distributions forces the system to be algebraic.

In this talk, we will be interested in the analog situation for discrete-time dynamics, that is for diffeomorphisms behaving like the time-one map of an Anosov flow, that are called *partially hyperbolic* – they contract a *stable distribution*, expand an *unstable distribution* and preserve a third *central* distribution. If these distributions are sufficiently regular and the sum stable-unstable is contact, they induce an invariant rigid geometric structure called *path structure* – or *Lagrangian-contact structure*. The latter can be studied in the framework of *Cartan geometries*, which reveals extremely fruitful to understand the rigidity of the studied diffeomorphisms.

We will present a classification result obtained in [MM21b] for such three-dimensional partially hyperbolic diffeomorphisms of contact type, and a work in progress with Elisha Falbel concerning higher-dimensional partially hyperbolic diffeomorphisms. If the time permits, we will address related results obtained in [FMMV21, MM21a] concerning path structures with large automorphism groups.

REFERENCES

- [BFL92] Yves Benoist, Patrick Foulon, and François Labourie. Flots d’Anosov à distributions stable et instable différentiables. *Journal of the American Mathematical Society*, 5(1):33–74, 1992.
- [FMMV21] E. Falbel, M. Mion-Mouton, and J. M. Veloso. Cartan connections and path structures with large automorphism groups. *International Journal of Mathematics*, 32(12):2140016, 2021.
- [Ghy87] Étienne Ghys. Flots d’Anosov dont les feuilletages stables sont différentiables. *Annales Scientifiques de l’École Normale Supérieure. Quatrième Série*, 20(2):251–270, 1987.
- [MM21a] Martin Mion-Mouton. Geometrical compactifications of geodesic flows and path structures. *arXiv:2112.02900 [math]*, 2021.
- [MM21b] Martin Mion-Mouton. Partially hyperbolic diffeomorphisms and Lagrangian contact structures. *Ergodic Theory and Dynamical Systems*, pages 1–47, 2021.