MAURER-CARTAN ELEMENTS: AN OPERADIC POINT OF VIEW

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Abstract:

1) I will introduce the notion of Maurer-Cartan elements as solutions to the Maurer-Cartan equation in associative, Lie, preLie, Lie admissible algebras and their versions up to homotopy. The description of their role in deformation theory will be explained. We will provide the audience with several examples like the Hochschild and the Chevalley-Eilenberg chain complexes, and more generally the convolution algebras coming from the operadic calculus. Finally, I will ask the question: how does the Maurer-Cartan equation appears conceptually ? I will provide a criterion on operads P such that the category of P-algebras admits such a Maurer-Cartan equation and can thus be twisted by Maurer-Cartan elements.
This is based on a joint work with V. Dotsenko and S. Shadrin available at
https://arxiv.org/abs/1810.02941.

2) The integration of the degree 0 part of a dg Lie algebra with the Baker-Campbell-Hausdorff formula gives rise to a gauge group which acts on the set of Maurer-Cartan elements. We will see how this integration process can be made more efficient when the Lie algebra is not generic and comes from stronger algebraic structures like preLie or Lie admissible structures. This is motivated by applications in operadic deformation theory and we will see how the three main functorial ways to create homotopy algebras can be achieved in this way under gauge group action.
This is based on a joint work with V. Dotsenko and S. Shadrin available at
https://arxiv.org/abs/1502.03280 and another one with Ricardo Campos to appear soon.

3) Lie theory is based on an equivalence between Lie algebras and Lie groups, but what happens higher up for homotopy Lie algebras? I will present a novel approach to the problem of integrating homotopy Lie algebras by representing the Maurer–Cartan space functor with a universal cosimplicial object. As an example of application, I will construct a coherent hierarchy of higher Baker–Campbell–Hausdorff formulas. I will conclude these talks by applying this theory to rational homotopy theory and show that the functors of this higher Lie theory recover the Bousfiled-Kan Q-completion and so capture faithfully the rational homotopy type of spaces.
This is based on a joint work with D. Robert-Nicoud to appear very soon.