

The homotopy type of associative and commutative algebras

Ricardo Campos
Institut Montpellierain Alexander Grothendieck,

Abstract

Given a topological space, how much of its homotopy type is captured by its algebra of singular cochains? The experienced rational homotopy theorist will argue that one should consider instead a commutative algebra of forms. This raises the more algebraic question "Given a dg commutative algebra, how much of its homotopy type (quasi-isomorphism type) is contained in its associative part?" Despite its elementary formulation, this question turns out to be very subtle. Surprisingly perhaps, it fits very well within the "Koszul duality & Operads" topics of this workshop. In this talk, I will show how one can use operadic deformation theory to give an affirmative answer in characteristic zero. We will also see how the Koszul duality between Lie algebras and commutative algebras allows us to use similar arguments to deduce that under good conditions Lie algebras are determined by the (associative algebra structure of) their universal enveloping algebras. (Joint with Dan Petersen, Daniel Robert-Nicoud and Felix Wierstra and based on arXiv:1904.03585)