

Mathematical Models for Chromonic Liquid Crystals

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Chromonic liquid crystals (CLCs) are lyotropic phases formed by certain dyes, drugs, and short nucleic-acid oligomers in aqueous solutions. Since many biological processes take place in such solutions, it is no wonder that CLCs promise to find an increasing number of applications in medical sciences. So far these materials have mainly been modeled within the classical Oseen-Frank elastic theory, with only the twist constant K_{22} appreciably smaller than both K_{11} and K_{33} , the splay and bend constants, respectively [1]. Moreover, by assuming that the saddle-splay constant K_{24} is greater than K_{22} , a typical *escaped-twist* (ET) ground state observed in capillary tubes could be justified within the classical theory, in good accord with the experimental data [2].

The problem with this success is that assuming

$$K_{24} > K_{22} \tag{1}$$

violates one of Ericksen's inequalities [3], which guarantee that the Oseen-Frank elastic free-energy functional is bounded below, a condition generally necessary for the existence of a minimizer. Eventually, the variational problem in cylindrical symmetry solved by the ET distortion turns out to be well-posed, because of the specific boundary conditions imposed on the nematic director on the cylinder's wall. But other paradoxical behaviors follow, if we insist on retaining (1).

Among these is the prediction that a chromonic droplet would disintegrate spontaneously in an isotropic environment, as it can be shown that splitting a droplet in two would save energy. To mend this and similar other paradoxes, the classical model is appropriately revisited and extended. The resulting new model confirms the results of the classical one while resolving its paradoxes, but still poses several other questions, which will be presented and briefly discussed at the end of the lecture.

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References

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