UNIFORM PROFILE NEAR THE POINT DEFECT OF LANDAU-DE GENNES MODEL IN THE VANISHING ELASTICITY LIMIT

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Abstract: For the simplified Landau-de Gennes model,

$$I_\varepsilon(Q,\Omega) := \int_\Omega \left\{ \frac{1}{2}|\nabla Q|^2 + \frac{1}{\varepsilon^2}(-\frac{a^2}{2}\mathrm{tr}(Q^2) - \frac{b^2}{3}\mathrm{tr}(Q^3) + \frac{c^2}{4}[\mathrm{tr}(Q^2)]^2 + C) \right\} dx,$$

the energy will enforce the uniaxial constraint $Q = s_+(n \times n - \frac{1}{3}\mathrm{Id})$ when $\varepsilon \to 0$. Previous results indicate that under suitable assumptions on the boundary condition $Q|_{\partial\Omega}$, the global minimizer Q_ε converges strongly in $H^1(\Omega)$ and $C^k_{loc}(\Omega \setminus \mathcal{S}(n_*))$ to a uniaxial minimizer $Q_* = s_+(n_* \otimes n_* - \frac{1}{3}\mathrm{Id})$ up to some subsequence, where $n_* \in H^1(\Omega, \mathbb{S}^2)$ is a minimizing harmonic map and $\mathcal{S}(n_*)$ denotes the singular set of n_*.

In this talk we further investigate the structure of minimizers Q_ε in the core of a point defect $x_0 \in \mathcal{S}(n_*)$ by studying the blow-up profile of $Q_{\varepsilon_n}(x_n + \varepsilon_n y)$ where x_n will be carefully chosen and converge to x_0. We show that $Q_{\varepsilon_n}(x_n + \varepsilon_n y)$ will converge in $C^2_{loc}(\mathbb{R}^n)$ to a tangent map $Q(x)$ which at infinity behaves like a "hedgehog" solution that coincides with the asymptotic profile of n_* near x_0. Such convergence result further implies that the minimizer Q_{ε_n} can be well approximated by the Oseen-Frank minimizer n_* outside the $O(\varepsilon_n)$ neighborhood of the point defect (such neighborhood can be regarded as the defect core). This is a joint work with Arghir Zarnescu.