

Generation of singularities from the initial datum for Hamilton-Jacobi equations

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Abstract

We consider the Cauchy problem

$$\begin{cases} \partial_t u(t, x) + H(t, x, D_x u(t, x)) = 0, & \text{in }]0, T[\times \Omega, \\ u(0, x) = u_0(x), & \text{in } \Omega. \end{cases}$$

where the Hamiltonian H is smooth and convex in $D_x u$, and the initial datum u_0 is merely Lipschitz continuous. We investigate how the small time regularity of u is related to the properties of u_0 . It is well known that the differentiability of u_0 at a given point is neither a necessary nor a sufficient condition for the differentiability of u in a neighbourhood. Here we characterize the regularity of a solution u near a point x_0 in terms of the properties of the proximal subdifferential of the initial data at x_0 . The main tool is a one-to-one correspondence between the elements of the proximal subdifferential and the forward classical characteristics emanating from x_0 . The results are in collaboration with P. Albano and P. Cannarsa