Viscous Hamilton-Jacobi equations in the superquadratic case

Alessio Porretta

University of Rome Tor Vergata

email: porretta@mat.uniroma2.it

Abstract

We discuss properties of the viscous Hamilton-Jacobi equation

$$\begin{cases} u_t - \Delta u = |Du|^p & \text{in } (0,\infty)\times\Omega, \\ u = 0 & \text{in } (0,\infty)\times\partial\Omega, \\ u(0) = u_0 & \text{in } \Omega, \end{cases}$$

in the super-quadratic case $p > 2$. Here Ω is a bounded domain in $\mathbf{R^N}$. In the super-quadratic regime, solutions may be continuous but with a gradient blow up; in this case the second order equation exhibits very peculiar phenomena. Some properties are similar to first order problems, such as loss of boundary conditions and appearance of singularities, but the presence of diffusion let singularities appear and disappear, in a very unusual way. In the talk I will present results obtained in collaboration with Philippe Souplet which describe the qualitative behavior of the solution, starting from smooth initial data. This includes the analysis of blow-up rates, blow-up profiles, life after blow-up, loss and recovery of boundary conditions.