

On the equivalence of viscosity solutions and distributional solutions for the time-fractional diffusion equation

Hiroyoshi Mitake

University of Tokyo

email: mitake@ms.u-tokyo.ac.jp

Abstract

In this talk, we consider an initial-boundary value problem for the time-fractional diffusion equation. We show the equivalence of two notions of weak solutions, viscosity solutions and distributional solutions. It is worth emphasizing that in general the notion of viscosity solutions is based on the comparison principle, while the notion of distributional solutions is based on the variational principle. Since two notions of weak solutions are introduced in totally different manners, it is highly nontrivial whether two notions are same or not.

In our approach, we use the discrete scheme for time-fractional diffusion equations which was introduced by Giga-Liu-Mitake (Asymptot. Anal. 2020). A main difficulty is in proving that the error term which comes from the approximated solution and the distributional solution converges to zero in a suitable weak sense. The idea to overcome this difficulty is to introduce an approximation of kernel in consideration of the discrete scheme. Due to the discrete scheme and kernel approximation, we can get the precise error estimate which enables us to get our main theorem.

This is a joint work with Y. Giga (U. Tokyo) and S. Sato (U. Tokyo).