

Characterization of F -concavity preserved by the Dirichlet heat flow

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Abstract

F -concavity is a generalization of power concavity and, actually, the largest available generalization of the notion of concavity. We characterize the F -concavities preserved by the Dirichlet heat flow in convex domains on \mathbb{R}^n with $n \geq 1$, and complete the study of preservation of concavity properties by the Dirichlet heat flow, started by Brascamp and Lieb in 1976 and developed in some recent papers. More precisely,

- (1) we discover the strongest F -concavity preserved by the Dirichlet heat flow;
- (2) we show that log-concavity is the weakest F -concavity preserved by the Dirichlet heat flow when $n \geq 2$ and otherwise except for quasi-concavity;
- (3) we prove that if F -concavity is strictly weaker than log-concavity and $n \geq 2$, then there exists an F -concave initial datum such that the corresponding solution to the Dirichlet heat flow loses any reminiscence of concavity.

Furthermore, we find a sufficient and necessary condition for F -concavity to be preserved by the Dirichlet heat flow.

This is a joint work with Paolo Salani (Università di Firenze) and Asuka Takatsu (Tokyo Metropolitan University).