Controllability and stabilization for a degenerate wave equation in non divergence form with drift

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Abstract

We consider the problem

$$\begin{cases} u_{tt} - a(x)u_{xx} - b(x)u_x = 0, & (t,x) \in Q, \\ u(t,0) = 0, & t \in [0,+\infty), \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in (0,1), \end{cases} \tag{1}$$

where $Q = (0,+\infty) \times (0,1)$, $f \in L^2_{\rm loc}[0,+\infty)$, $a, b \in C^0[0,1]$, $a > 0$ on $(0,1]$ and $a(0) = 0$. At $x = 1$ we consider different boundary conditions according to the considered problem. If we are interested in a controllability problem (see [2]), we assume

$$u(t,1) = f(t), \quad t \in [0,+\infty);$$

thus the function f acts as a boundary control and it is used to drive the solution to 0 at a given time T.

If we are interested in the stabilization problem (see [3]) we consider as a boundary condition the following damping one

$$u_t(t,1) + \eta u_x(t,1) + \beta u(t,1) = 0, \quad t \in [0,+\infty),$$

where η is a given function and β is a nonnegative constant. Clearly the presence of the drift term and the nondivergence form lead to a different setting with respect to the one in [1] and they give rise to some new difficulties. However, thanks to some suitable assumptions on the drift term, one can prove some estimates on the associated energy that are crucial to drive the solution to 0 at time T or to obtain a uniform exponential decay as $t \to +\infty$.

References

[1] F. Alabau-Boussouira and P. Cannarsa and G. Leugering, Control and stabilization of degenerate wave equations, *SIAM J. Control Optim.*, **55** (2017), pp. 2052-–2087.

[2] I. Boutaayamou, G. Fragnelli, D. Mugnai, *Boundary controllability for a degenerate wave equation in non divergence form with drift*, submitted.

[3] G. Fragnelli, D. Mugnai, *Linear stabilization for a degenerate wave equation in non divergence form with drift*, preprint.