A Generalized Mean Field Game Model for the Dynamics of Pedestrians with Limited Predictive Abilities

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Abstract

This paper investigates the model for pedestrian flow firstly proposed in [Cristiani et al., DOI:10.1137/140962413]. The model assumes that each individual in the crowd moves in a known domain, aiming at minimizing a given cost functional. Both the pedestrian dynamics and the cost functional itself depend on the position of the whole crowd. In addition, pedestrians are assumed to have predictive abilities, but *limited in time*, extending only up to θ time units into the future, where $\theta \in [0, \infty)$ is a model parameter.

1) For $\theta = 0$ (no predictive abilities), we recover the modeling assumptions of the Hughes's model, where people take decisions on the basis of the current position of the crowd only.

2) For $\theta \to \infty$, instead, we recover the standard mean-field game (MFG) setting, where people are able to forecast the behavior of the others at any future time and take decisions on the basis of the current and future position of the whole crowd.

3) For very short values of θ (typically coinciding with a single time step in a discrete-in-time setting), we recover instead the MFG setting joined to the instantaneous model predictive control technique.

4) For intermediate values of θ we obtain something different: as in the Hughes's model, the numerical procedure to solve the problem requires to run an off-line procedure at any fixed time t, which returns the current optimal velocity field at time t by solving an associated backward-in-time Hamilton–Jacobi–Bellman equation; but, differently from the Hughes's model, here the procedure involves a prediction of the crowd behavior in the sliding time window $[t, t + \theta)$, therefore the optimal velocity field is given by the solution to a forward-backward system which joins a Fokker–Planck equation with a Hamilton–Jacobi–Bellman equation as in the MFG approach. The fact that a different forward-backward system must be solved at any time t arises new interesting theoretical questions. Numerical tests will give some clues about the well-posedness of the problem.

Reference paper: E. Cristiani, A. De Santo, M. Menci, *A generalized mean-field game model for the dynamics of pedestrians with limited predictive abilities*, Communications in Mathematical Sciences, in press.