Bilinear control of PDE's par scalar-input control and applications

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Abstract

In this talk, we present some recent results on the bilinear control of abstract first order PDE's in view of applications to parabolic PDE's. Scalar-input bilinear control means that the control takes the form $p(\cdot)B$ where p is a real-valued function depending only on time, whereas B is a given fixed control operator. Hence the only part which is used to control the given abstract equation from an initial state to a desired target is the control function p. Such control systems are nonlinear since $p(\cdot)B$ acts on the state u solution of the controlled system.

This limited action of the control is suitable to application in quantum control, or for smart materials which are modifying their physical characteristics (the state) under the action of a control. Such control systems present a well-known drawbacks in infinite dimensional frameworks, since when B is bounded it can be shown that the set of reachable states has a dense complement in the usual functional frame (Ball , Marsden and Slemrod 1982). Several positive results were proved since then on dispersive equations, quantum equations, beam or wave equations... Comparatively, very little is known on parabolic equations.

We present some of our recent results on the stabilizability at high speed, and local, semi-global exact controllability results on the bilinear control of abstract first order equations, with applications to parabolic equations (joint works with P. Cannarsa and C. Urbani). We also present a new and original mathematical methodology to construct explicit infinite classes of control operators B that satisfy the necessary and sufficient conditions required for scalar-input controllability to hold. This gives direct concrete results for the bilinear control on heat equations, but also on dispersive or hyperbolic equations as well as for different examples of boundary conditions (joint work with C. Urbani).