Equivariant Traces for an Algebra of Fourier Integral Operators on $\mathbb{R}^n$

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We consider the algebra $\mathscr{B}$ of all operators on $\mathscr{S}(\mathbb{R}^n)$ given as finite sums

$$B = \sum R_g T_w A,$$

where

- A is a pseudodifferential operator in the Shubin calculus on $\mathbb{R}^n$
- For $w = a - ik \in \mathbb{C}^n$, T_w is the Heisenberg-Weyl operators given by $T_w u(x) = e^{ikx-iak/2}u(x-a)$, $u \in L^2(\mathbb{R}^n)$
- $g \mapsto R_g$ represents $g \in U(n)$ as a metaplectic operator, using the identification $\mathbb{C}^n \cong T^*\mathbb{R}^n$.

It is a consequence of Egorov's theorem for metaplectic operators that this is indeed an algebra.

Choosing an auxiliary operator such as $H = |x|^2 - \Delta$, we obtain expansions for

- $\mathrm{Tr}(B(H-\lambda)^{-K})$ in powers of λ and $\log\lambda$ for K large, as $\lambda \to \infty$ in a sector of $\mathbb{C}$,
- $\mathrm{Tr}(Be^{-tH})$ as $t \to 0^+$ in powers of t and $\log t$, and
- the pole structure of the meromorphic extension of $\zeta_B(z) = \mathrm{Tr}(BH^{-z})$.

Moreover, we find a noncommutative residue that extends the Wodzicki residue to this situation. For a discrete subgroup G of $\mathbb{C} \rtimes U(n)$ this allows us to define equivariant traces on $\mathscr{B}$.

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