

THE KOLMOGOROV-ARNOLD THEOREM REVISITED

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Abstract: There is a longstanding debate whether the Kolmogorov-Arnold representation theorem can explain the use of more than one hidden layer in neural networks. The Kolmogorov-Arnold representation decomposes a multivariate function into an interior and an outer function and therefore has indeed a similar structure as a neural network with two hidden layers. But there are distinctive differences. One of the main obstacles is that the outer function depends on the represented function and can be wildly varying even if the represented function is smooth. We derive modifications of the Kolmogorov-Arnold representation that transfer smoothness properties of the represented function to the outer function and can be well approximated by ReLU networks. It appears that instead of two hidden layers, a more natural interpretation of the Kolmogorov-Arnold representation is that of a deep neural network where most of the layers are required to approximate the interior function.