

# ON LOWER BOUNDS FOR THE BIAS-VARIANCE TRADE-OFF

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Abstract: It is a common phenomenon that for high-dimensional and nonparametric statistical models, rate-optimal estimators balance squared bias and variance. Although this balancing is widely observed, little is known whether methods exist that could avoid the trade-off between bias and variance. We propose a general strategy to obtain lower bounds on the variance of any estimator with bias smaller than a prespecified bound. This shows to which extent the bias-variance trade-off is unavoidable and allows to quantify the loss of performance for methods that do not obey it. The approach is based on a number of abstract lower bounds for the variance involving the change of expectation with respect to different probability measures as well as information measures such as the Kullback-Leibler or  $\chi^2$ -divergence. Some of these inequalities rely on a new concept of information matrices. In a second part, the abstract lower bounds are applied to several statistical models including the Gaussian white noise model, a boundary estimation problem, the Gaussian sequence model and the high-dimensional linear regression model. For these specific statistical applications, different types of bias-variance trade-offs occur that vary considerably in their strength. For the trade-off between integrated squared bias and integrated variance in the Gaussian white noise model, we propose to combine the general strategy for lower bounds with a reduction technique. This allows us to reduce the original problem to a lower bound on the bias-variance trade-off for estimators with additional symmetry properties in a simpler statistical model. To highlight possible extensions of the proposed framework, we moreover briefly discuss the trade-off between bias and mean absolute deviation.