

BERNSTEIN-VON MISES THEOREM FOR THE SCALE HYPERPARAMETER IN INVERSE PROBLEMS WITH A GAUSSIAN PRIOR

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Abstract: We study linear inverse problems from a Bayesian perspective with a Gaussian prior on the unknown function with mean 0 and covariance operator μG^{-2} . Using an adaptive hyperprior distribution on the scale parameter μ , we show that for mildly ill-posed inverse problems over Sobolev spaces, the corresponding posterior distribution of the unknown function with the plugged-in empirical Bayes estimator of the scale μ contracts at the optimal (minimax) rate in L^2 distance. We also show that for an appropriate hyperprior, the marginal posterior distribution of the scale parameter μ is approximately Gaussian, and discuss its connection to the Bernstein-von Mises phenomenon.