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Title : Maps of unfixed genus and blossoming trees

4-regular planar maps can be counted bijectively via two constructions: one with blossoming trees due to Schaeffer, and one with well-labeled trees due to Cori-Vauquelin-Schaeffer. As shown by Bouttier, Di Francesco, and Guitter, these bijections imply that the so-called 2-point function $R_i(g)$ of 4-regular planar maps (generating function of 4-regular maps with two points at dual distance smaller than i) satisfies the recurrence $R_i(g)=1+R_i(g)*(R_{i-1}(g)+R_i(g)+R_{i+1}(g))$ (with boundary condition $R_0(g)=0$), which remarkably can be solved explicitly.

I will show how the construction based on blossoming trees can be extended to 4-regular maps of unfixed genus. This provides a bijective proof of the fact that the corresponding counting series is expressed as the first term $r_1(g)$ of a sequence of series $(r_i(g))_{i\geq 1}$ that are now related by the recurrence $r_i(g)=i+r_i(g)*(r_{i-1}(g)+r_i(g)+r_{i+1}(g))$, with boundary condition $r_0(g)=0$ (this is the same recurrence as for the $R_i(g)$, except for the constant term i instead of 1), an expression that had previously been obtained via the configuration model and matrix integrals. Our construction also contains the one for 4-regular planar maps, and it explains the similarity between the series expressions for the planar case and the unfixed genus case.

This is joint work with Emmanuel Guitter.