DUNKL–SCHRÖDINGER OPERATORS WITH REVERSE HÖLDER CLASS POTENTIALS IN THE DUNKL SETTING

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Abstract

The aim of this talk is to study Dunkl–Schrödinger operator

$$L = -\Delta_k + V \text{ on } \mathbb{R}^N,\ N \geq 1,$$

where Δ_k is the Dunkl Laplacian and V is a non-negative polynomial (in particular, we can set $V(\mathbf{x}) = \|\mathbf{x}\|^2$) or, more generally, V belongs to the reverse Hölder class $\mathrm{RH}^q(dw)$ associated with $dw(\mathbf{x}) = \prod_{\alpha\in R} |\langle \alpha, \mathbf{x}\rangle|^{k(\alpha)} d\mathbf{x}$, that is,

$$\Big(\frac{1}{w(B)}\int_B V(\mathbf{y})^q\, dw(\mathbf{y})\Big)^{1/q} \leq \frac{C}{w(B)}\int_B V(\mathbf{y})\, dw(\mathbf{y}) \quad \text{for all Euclidean balls } B$$

and for some reasonable exponent $q > 1$. We define an auxiliary function

$$m(\mathbf{x})^{-1} = \sup\Big\{r > 0 : \frac{r^2}{w(B(\mathbf{x}, r))}\int_{B(\mathbf{x},r)} V(\mathbf{y})\, dw(\mathbf{y}) \leq 1\Big\}.$$

In the classical case, the function m was introduced by Ch. Fefferman ([1]).

In the first part of the talk we will discuss some properties of the classical Fefferman's auxiliary function m and the classical Fefferman–Phong inequality

$$\int_{\mathbb{R}^N} |f(\mathbf{x})|^2 m(\mathbf{x})^2\, d\mathbf{x} \leq C\langle Lf, f\rangle_{L^2(d\mathbf{x})}$$

([1, p. 146], see also Shen [4], [5, Lemma 1.9]). Then we will study their counterparts in the rational Dunkl setting.

The second part of the talk will be devoted to applications of the Fefferman–Phong inequality in the Dunkl setting. First, we utilize the Fefferman–Phong inequality in order to obtain lower and upper estimates for the number of eigenvalues of L which are less than the given number $\lambda > 0$. Further, again our important tool - the Fefferman-Phong inequality - will be used to provide the maximal and atomic characterizations of the Hardy spaces H^1_L associated with the operator L. The talk is based on [2] and [3].

References

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