SELECTED RESULTS IN REAL HARMONIC ANALYSIS IN THE RATIONAL DUNKL SETTING

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Abstract

The goal of the talk is to present selected results in real harmonic analysis in the rational Dunkl setting. We shall start by deriving estimates for the generalized translations

$$\tau_{\mathbf{x}} f(-\mathbf{y}) = c_k^{-1} \int_{\mathbb{R}^N} E(\mathbf{x}, i\xi) E(\mathbf{y}, -i\xi) \mathcal{F} f(\xi)\, dw(\xi)$$

of certain radial and non-radial functions f on $\mathbb{R}^N$, including estimates for the integral kernel of the heat Dunkl semigroup. Here $dw(\mathbf{x}) = \prod_{\alpha \in R} |\langle \alpha, \mathbf{x} \rangle|^{k(\alpha)}\, d\mathbf{x}$ denotes the associated measure, $E(\mathbf{x}, \mathbf{y})$ is the Dunkl kernel, and $\mathcal{F} f(\xi) = c_k^{-1} \int_{\mathbb{R}^N} f(\mathbf{x}) E(-i\xi, \mathbf{x}) f(\mathbf{x})\, dw(\mathbf{x})$ is the Dunkl transform. The obtained estimates will be given by means of the distance $d(\mathbf{x}, \mathbf{y})$ of the orbit of $\mathbf{x}$ to the orbit of $\mathbf{y}$ under the action of the reflection group G, that is,

$$d(\mathbf{x}, \mathbf{y}) = \min_{\sigma \in G} \|\sigma(\mathbf{x}) - \mathbf{y}\|,$$

the Euclidean distance $\|\mathbf{x} - \mathbf{y}\|$, and dw-volumes of the Euclidean balls and they will be in the spirit of estimates needed in real harmonic analysis on spaces of homogeneous type.

Then, if time permits, we shall discuss selected results, parallel to classical ones, which are proved by utilizing the obtained estimates for the generalized translation. In particular, we will be interested in:

- boundedness of maximal functions on various function spaces,
- characterizations of the real Hardy space H^1 in the Dunkl setting,
- boundedness of the Dunkl transform multiplier operators,
- boundedness of singular integral operators,
- upper and lower bounds for Littlewood-Paley square functions.

The results are joint works with Jean-Philippe Anker and Agnieszka Hejna.

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