Horospherical random graphs

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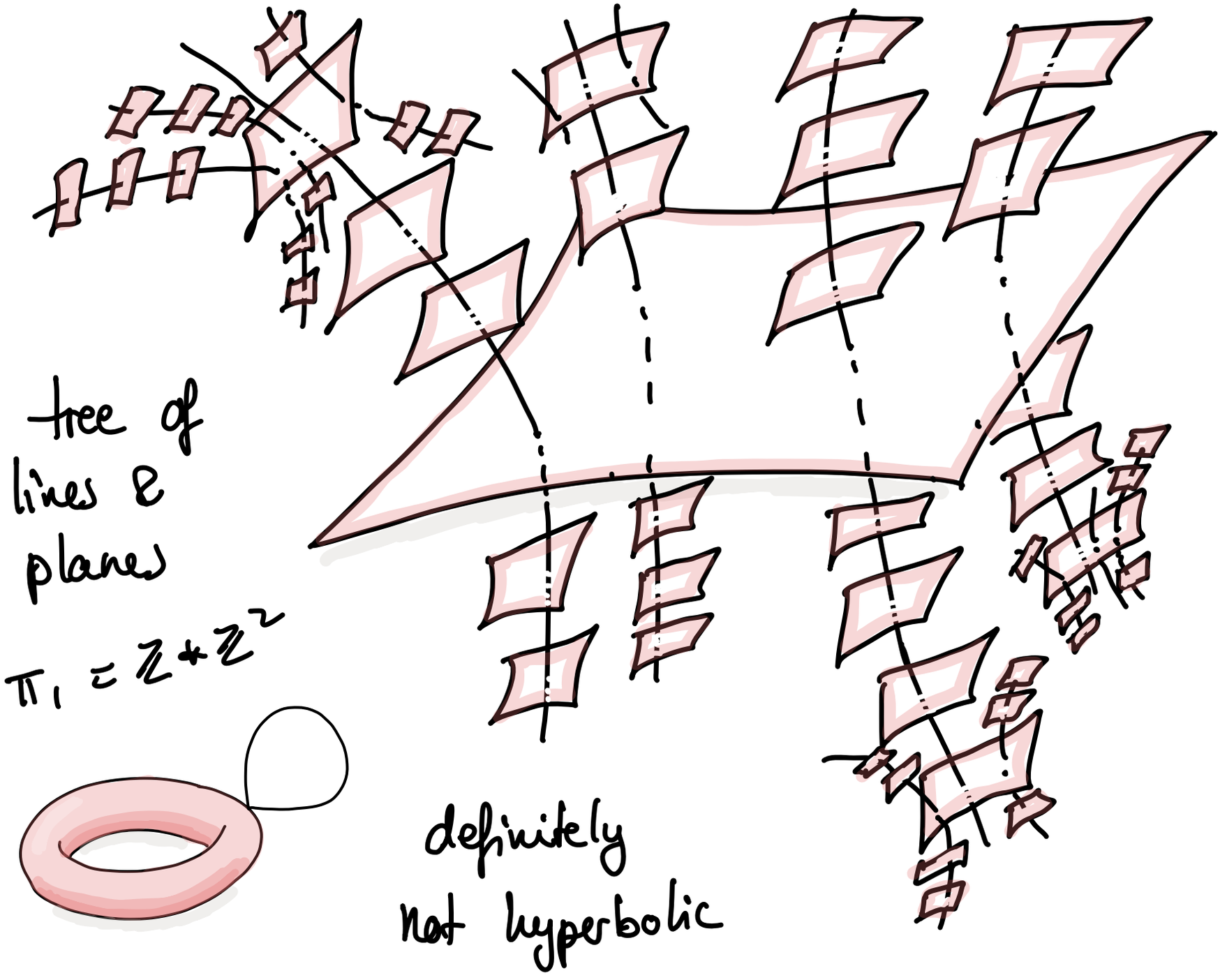
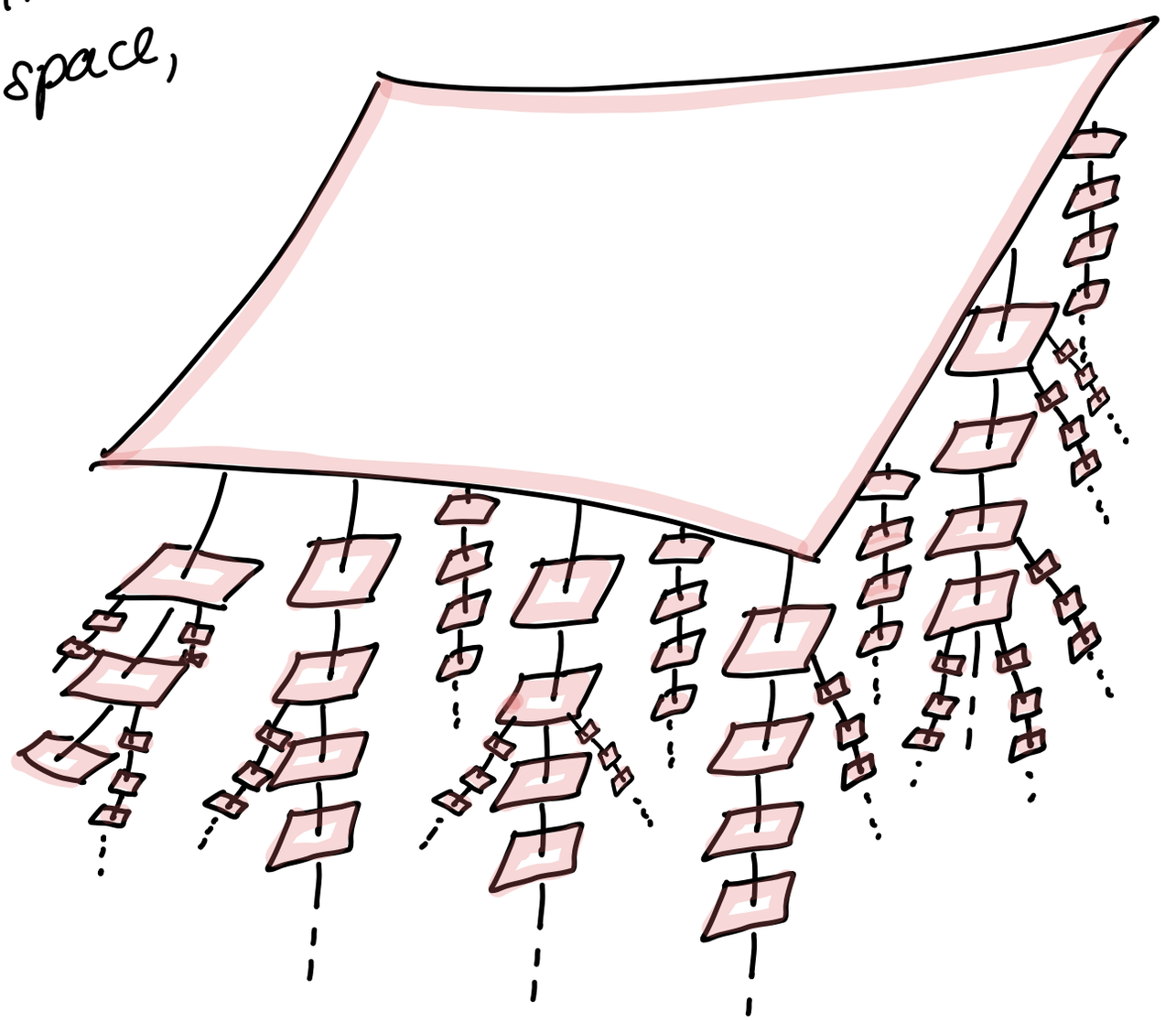
&

Austin Lawson

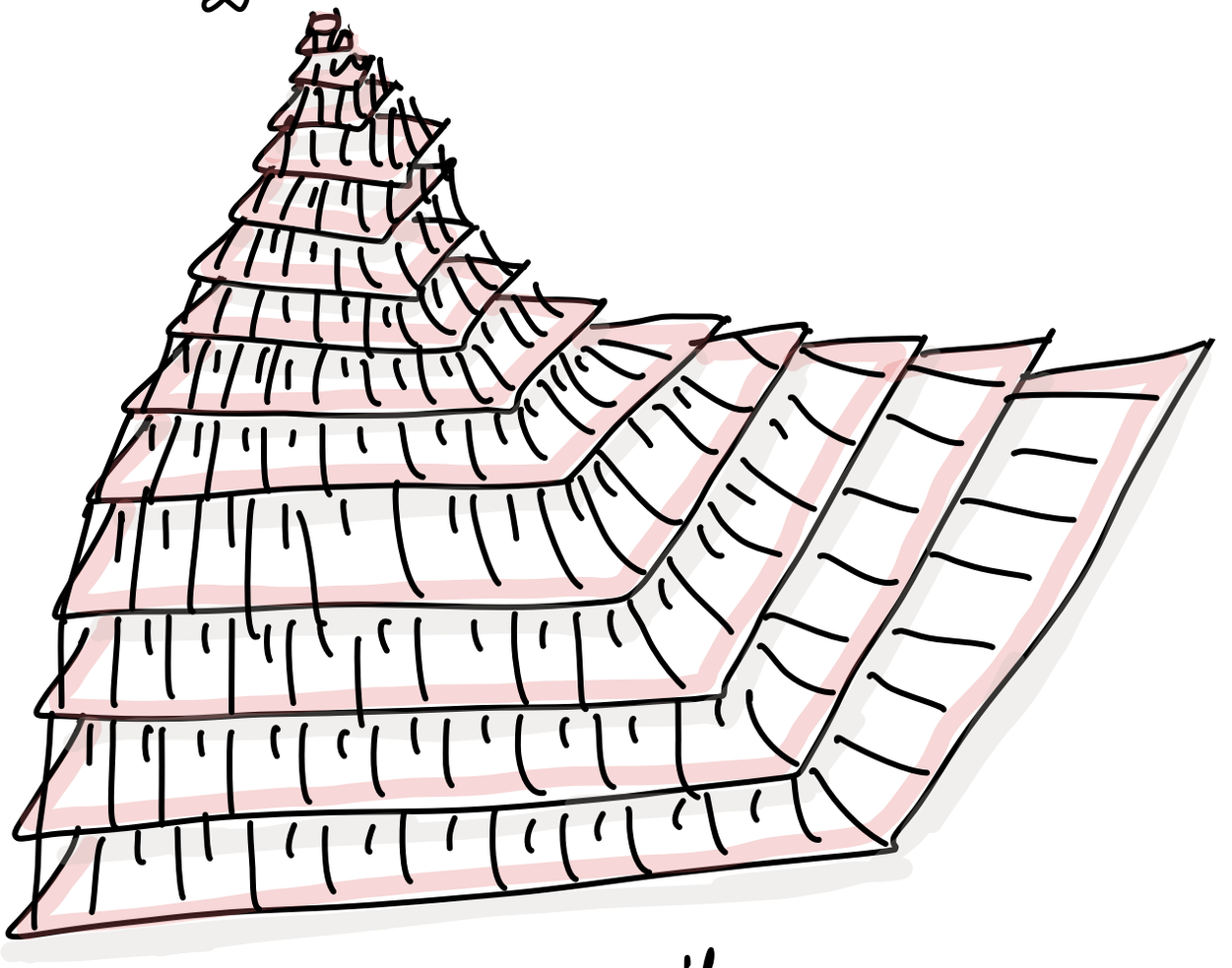

tree of lines & planes
$\pi_1 \cong \mathbb{Z} * \mathbb{Z}^2$
definitely not hyperbolic

BUT

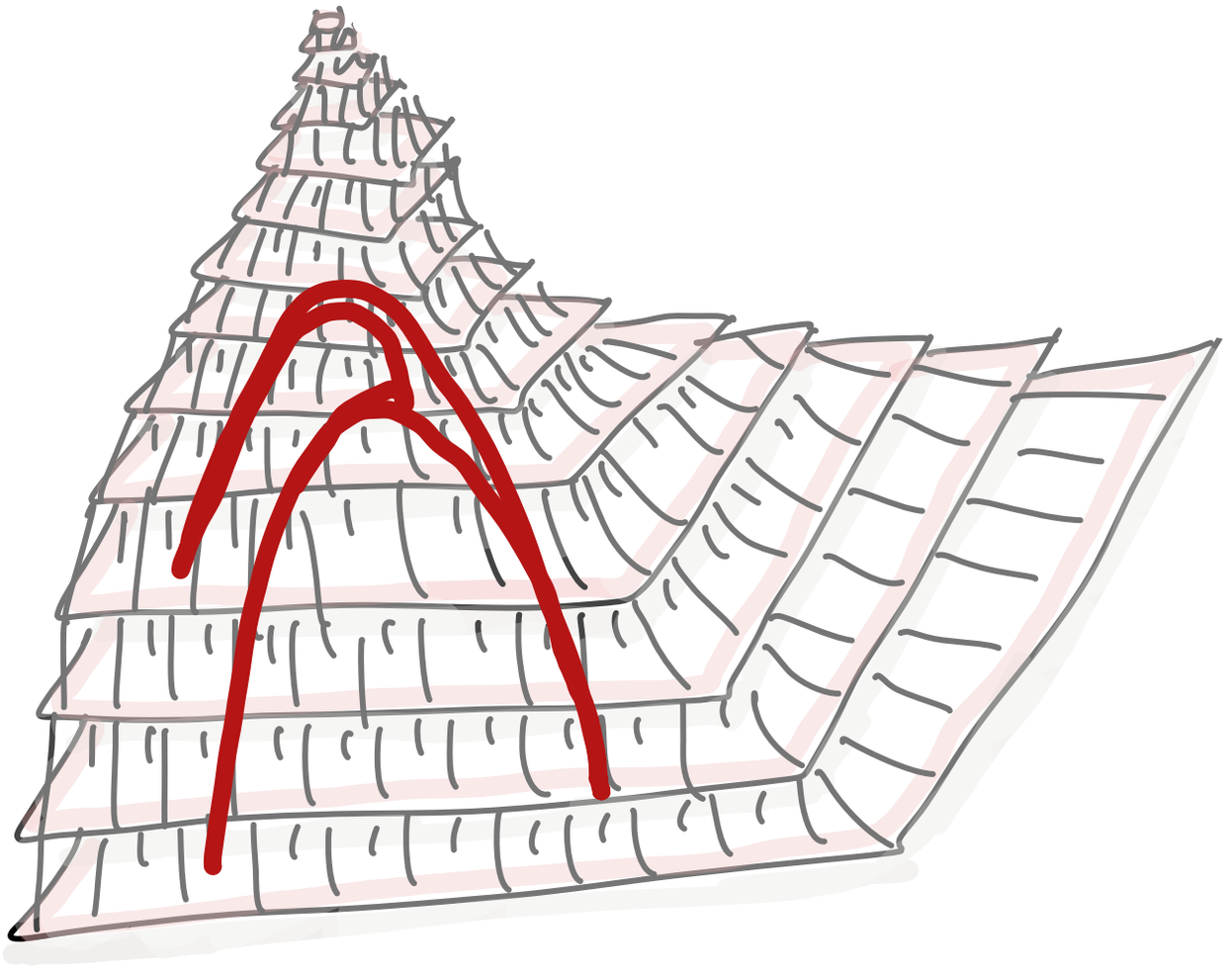
can embed it in a hyperbolic space, as follows



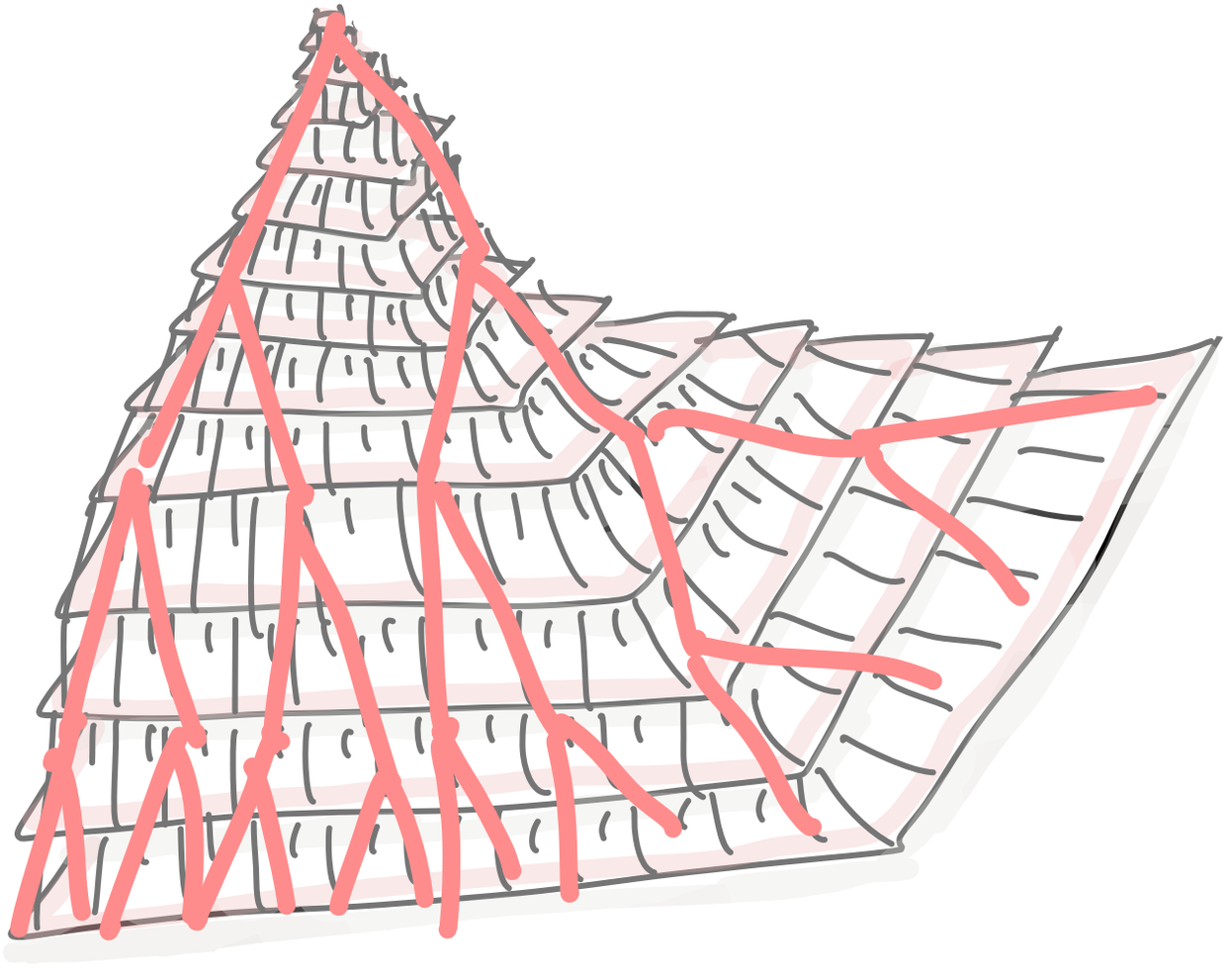
$\mathbb{Z}^2$ with metric $\frac{d}{2^n}$ at height n

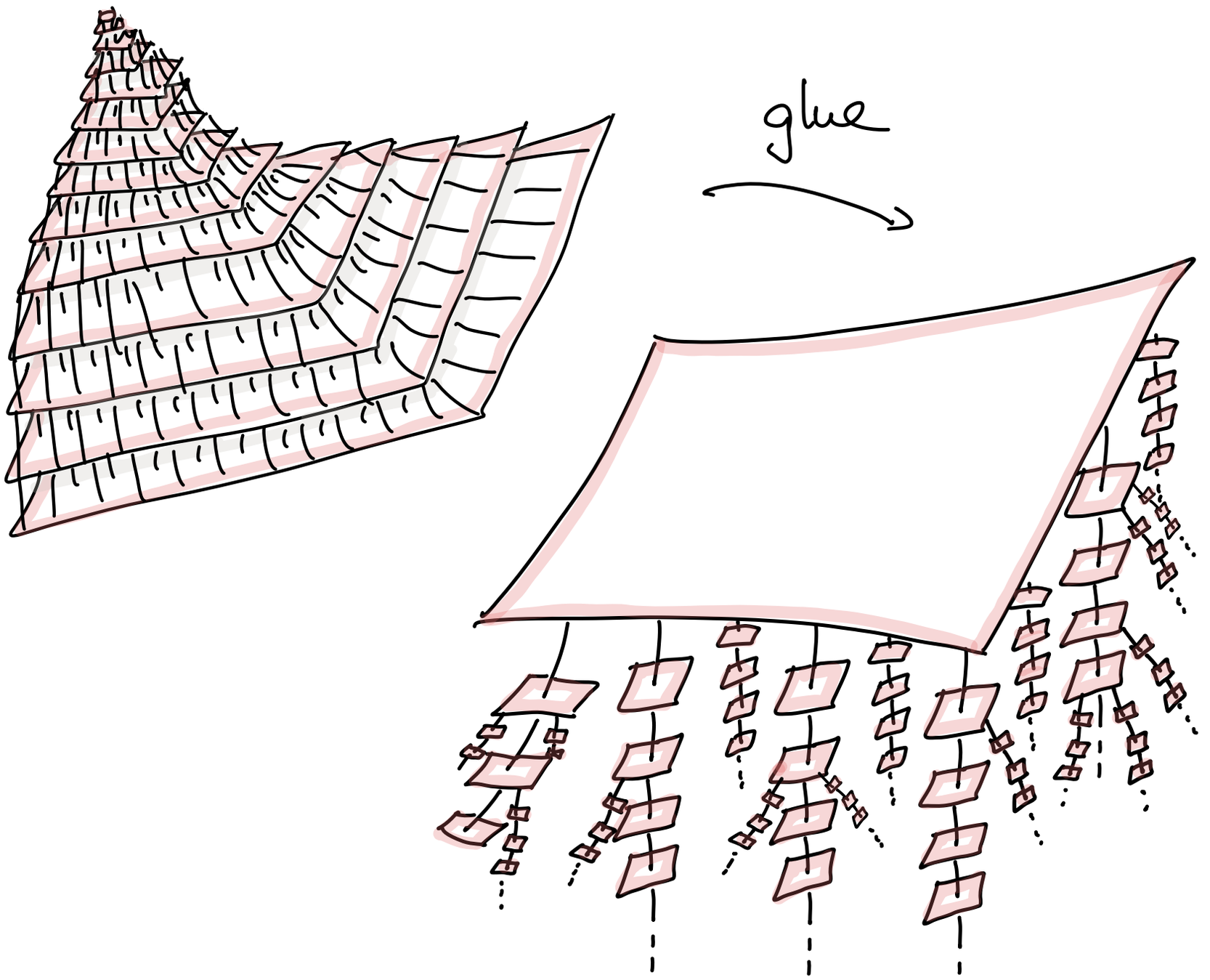


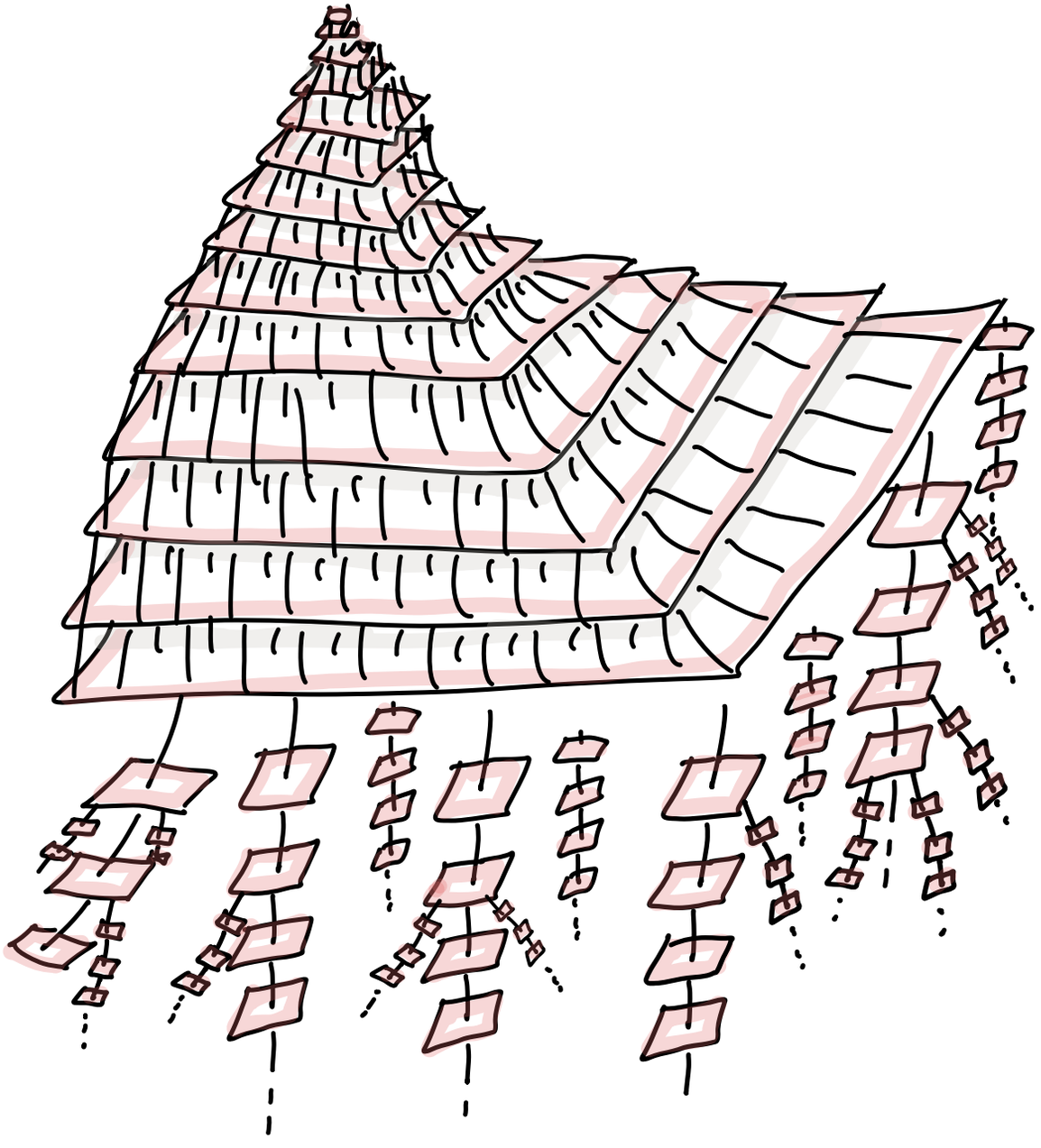
geodesics go up in the cusp

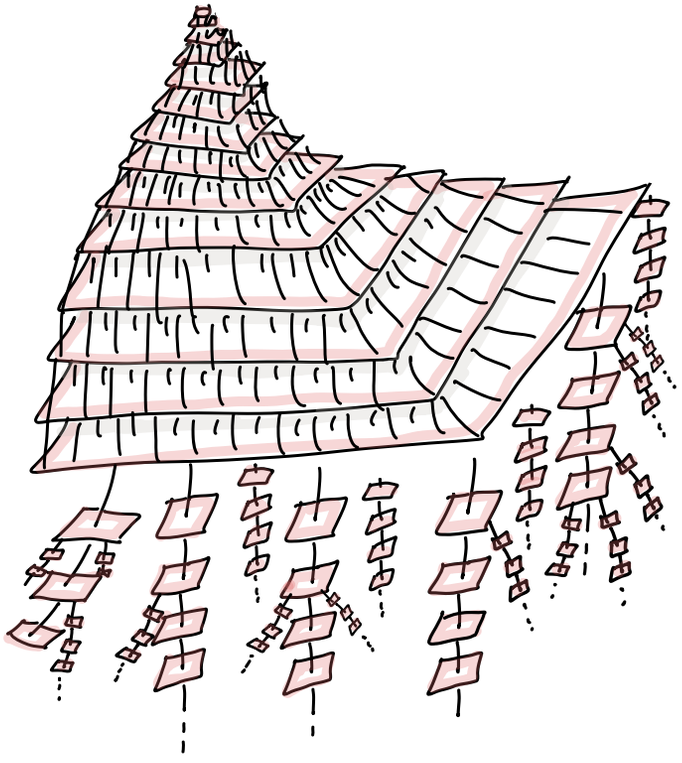


and triangles are thin

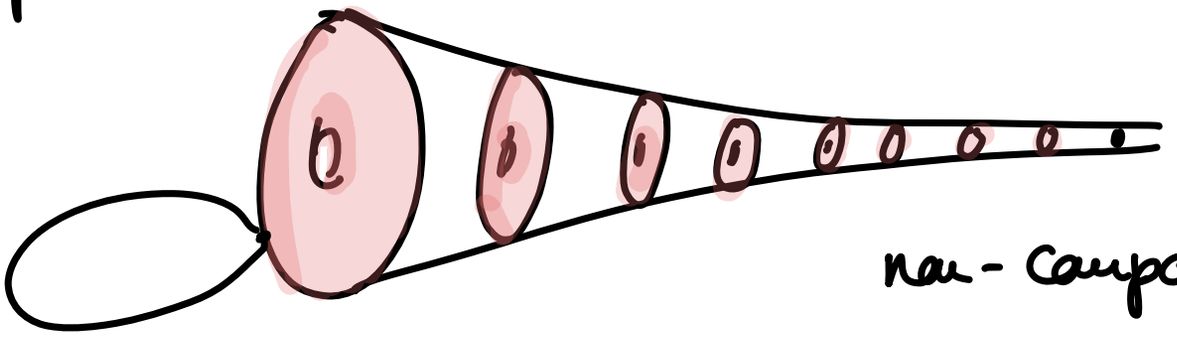



glue



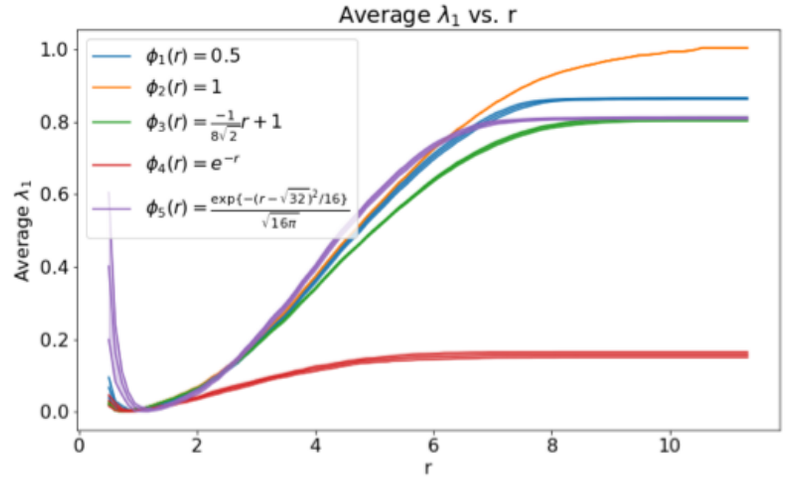


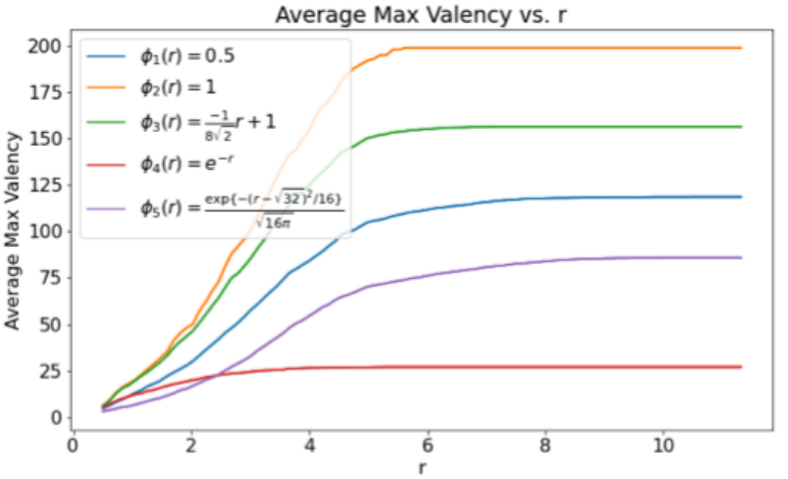
& quotient by $\mathbb{Z} * \mathbb{Z}^2$ to see

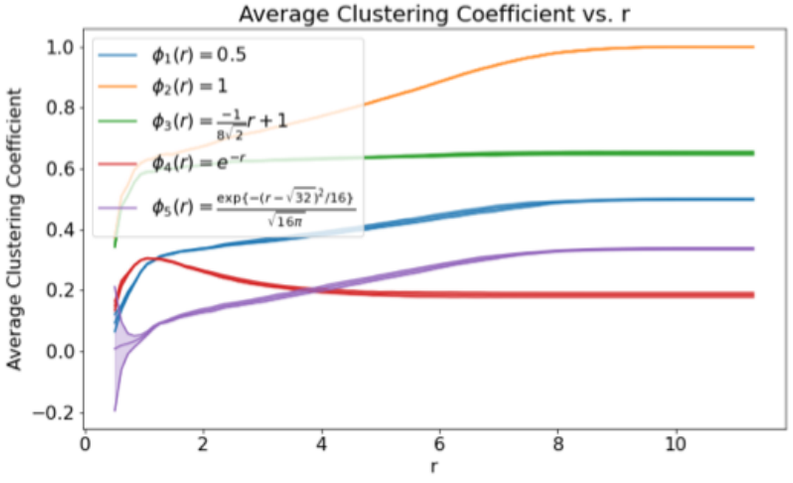


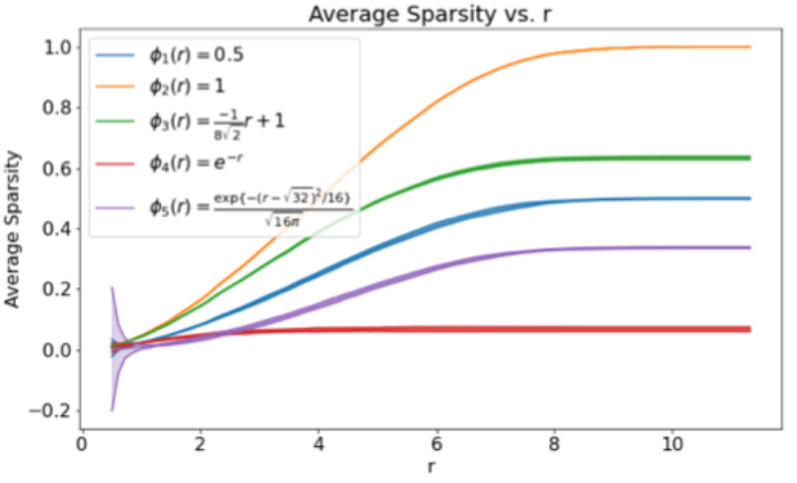
non-compact.

Soft (horospherical) graphs on a uniform distribution

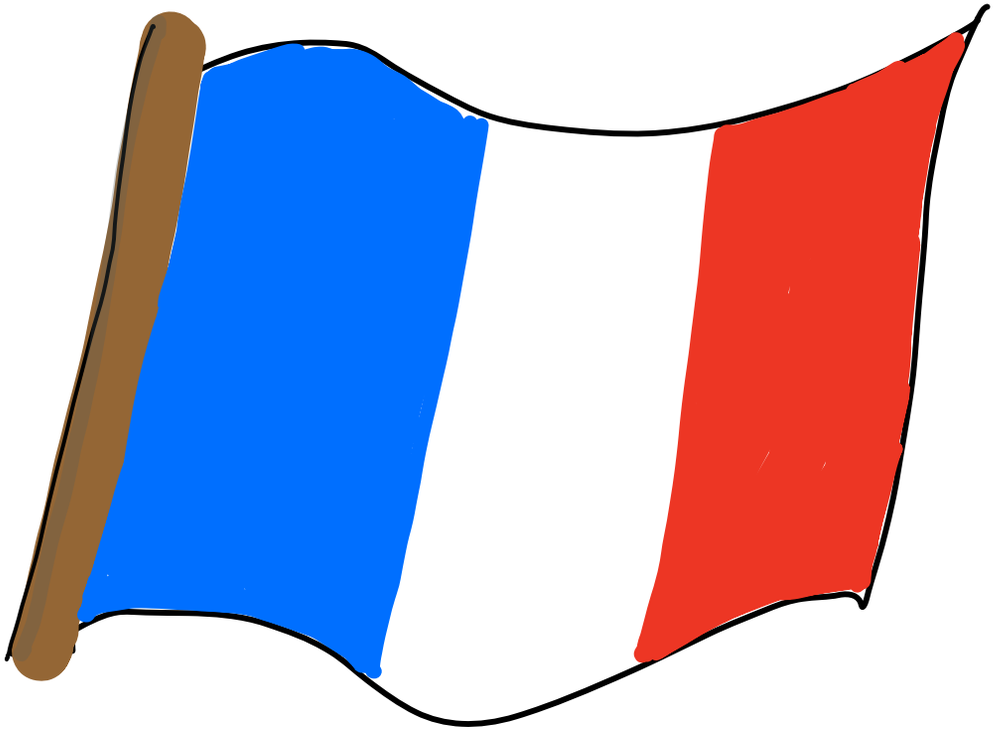




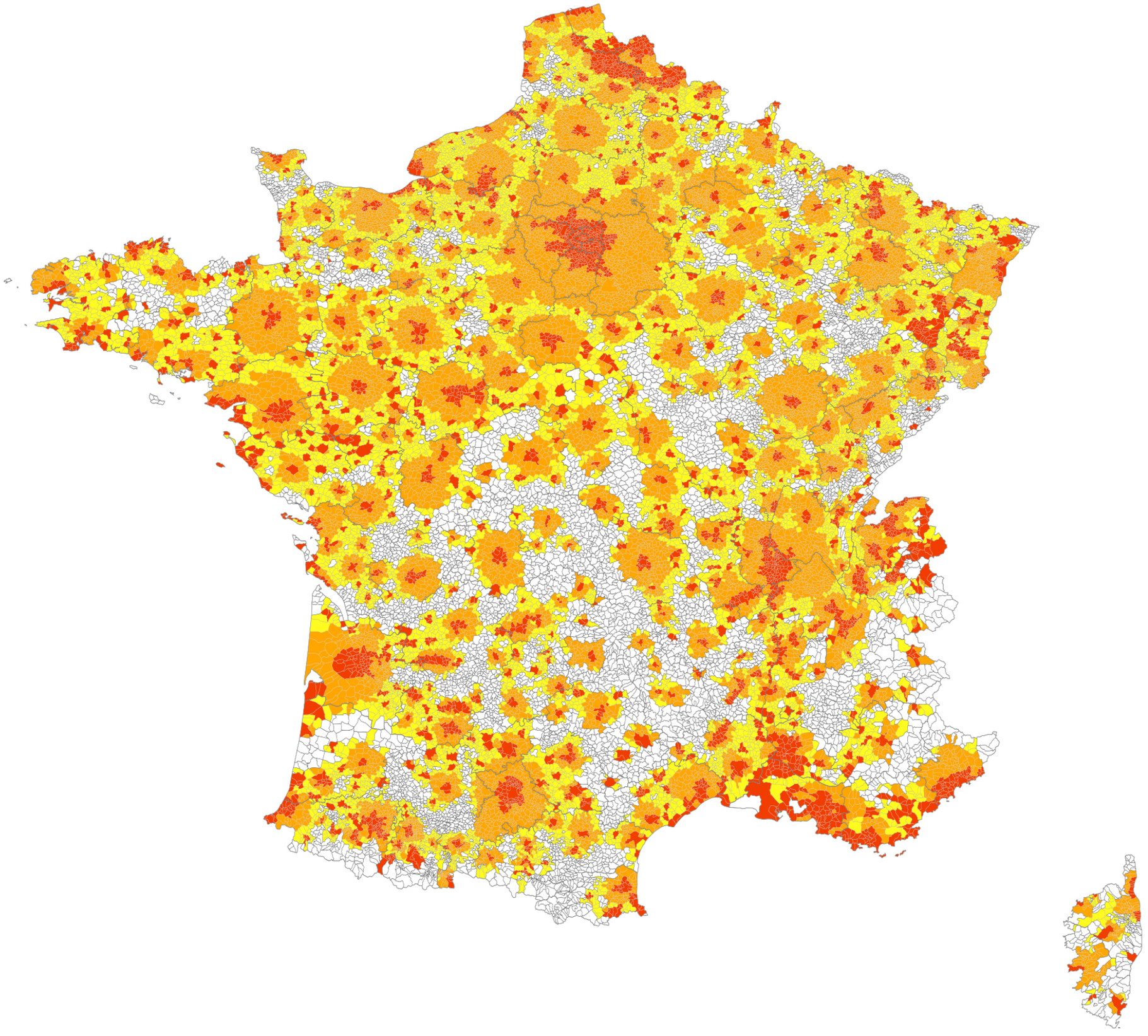


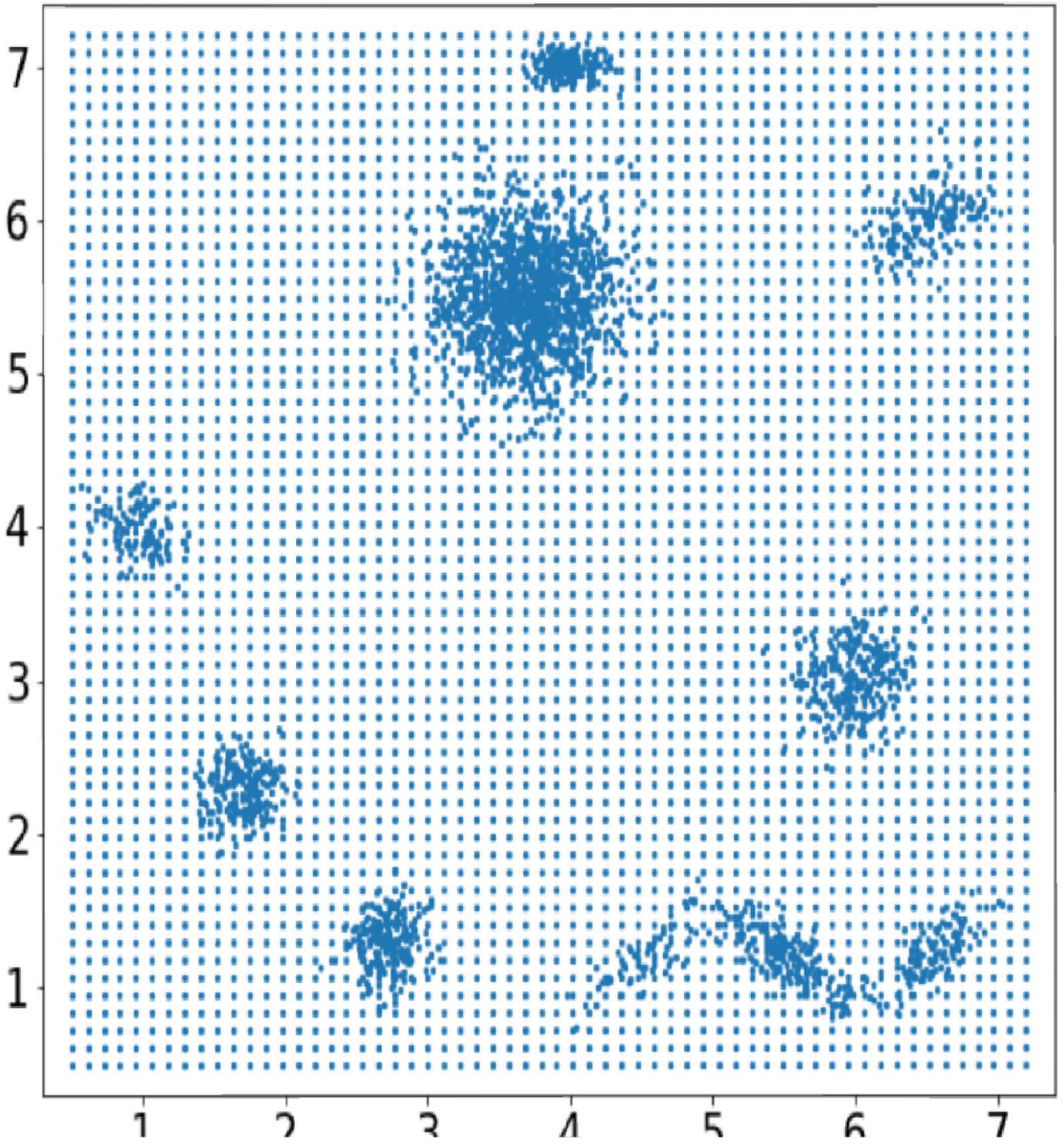


Experiment :



Horospherical graphs based in France




7
6
5
4
3
2
1
1
2
3
4
5
6
7

$$\phi(r) = \begin{cases} 1 & r \leq 0.3 \\ 0 & \text{otherwise} \end{cases} \qquad P(r) = \begin{cases} 1 & r \leq 0.15 \\ 0.1 & 0.15 < r \leq 0.3 \\ 0.05 & 0.3 < r \leq 1 \\ 0.01 & 0.15 < r \leq 3 \\ 0.005 & 3 < r \leq 10 \end{cases}$$

	Γ	G
λ_1	0.00035	0.29
Sparsity	0.015	0.016
Max Valency	470	149
Average Valency	97.1	27.2

TABLE 1. Various statistics for Γ and G

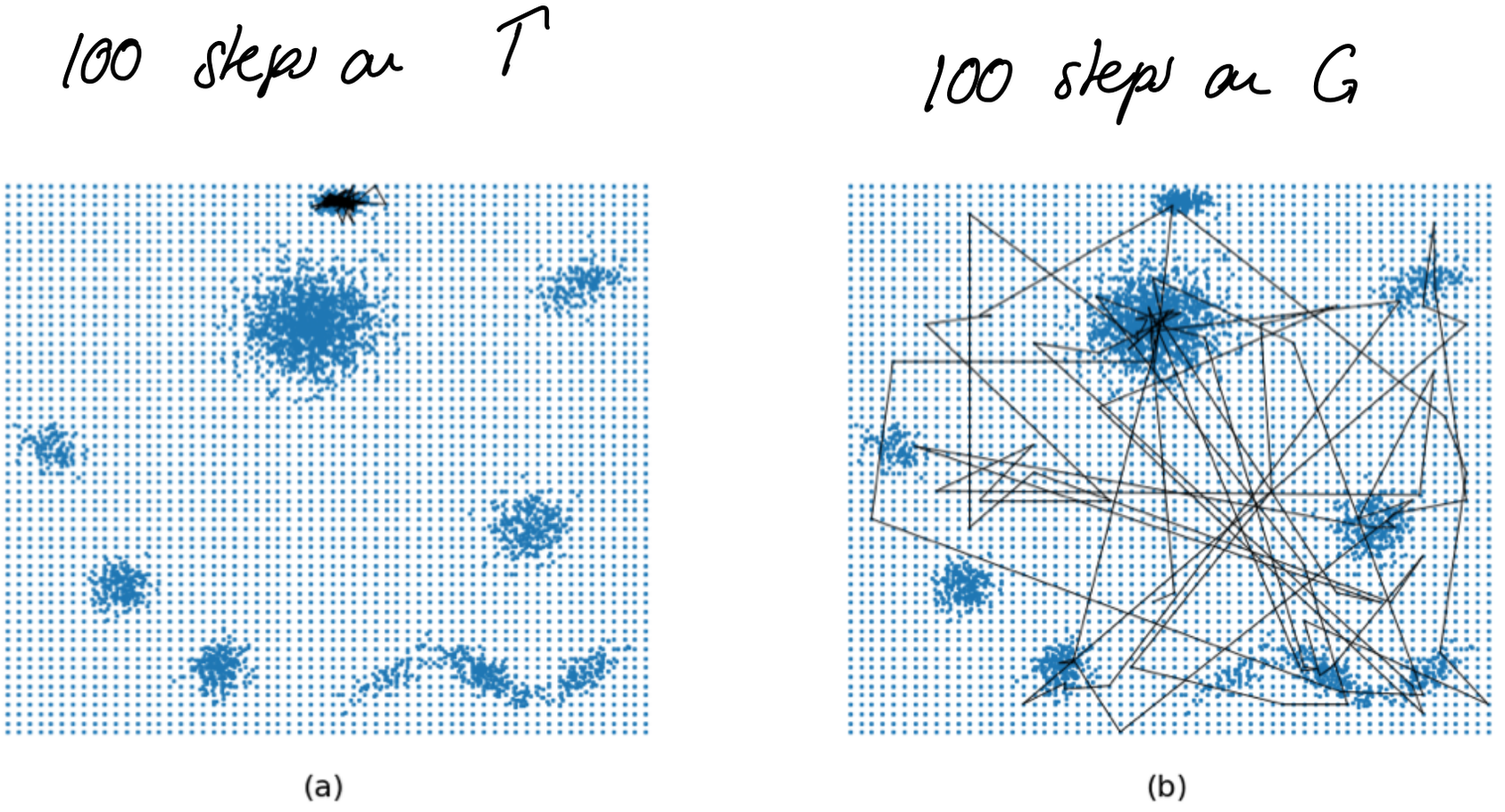


FIGURE 3. A simple random walk for Γ and G.

Here the walk replicates every 10 steps visiting a node not replicated before

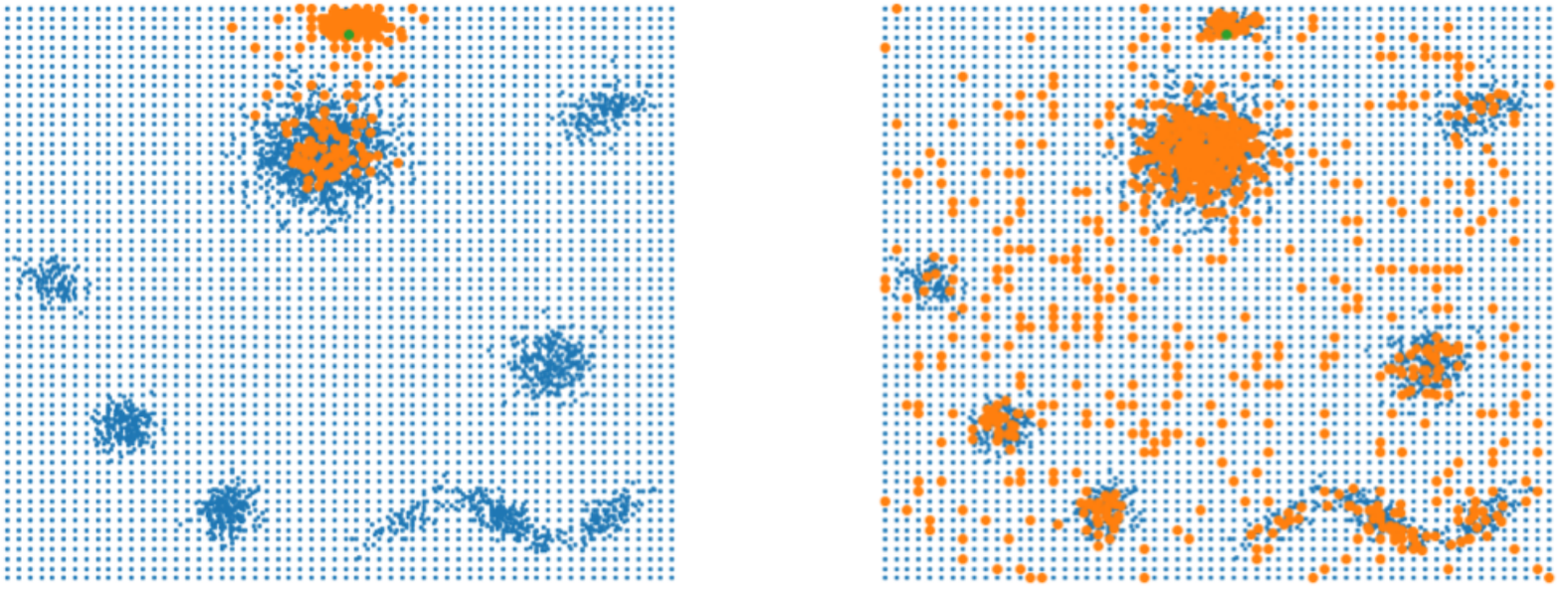


FIGURE 4. The initial positions of the replicants for the replicating random walk simulation on Γ and G.

If we reduce more long distance travel, replacing

$$P(r) = \begin{cases} 1 & r \leq 0.15 \\ 0.1 & 0.15 < r \leq 0.3 \\ 0.05 & 0.3 < r \leq 1 \\ 0.01 & 0.15 < r \leq 3 \\ 0.005 & 3 < r \leq 10 \end{cases}$$

$\rightarrow 0.05$

$\rightarrow 0.005$

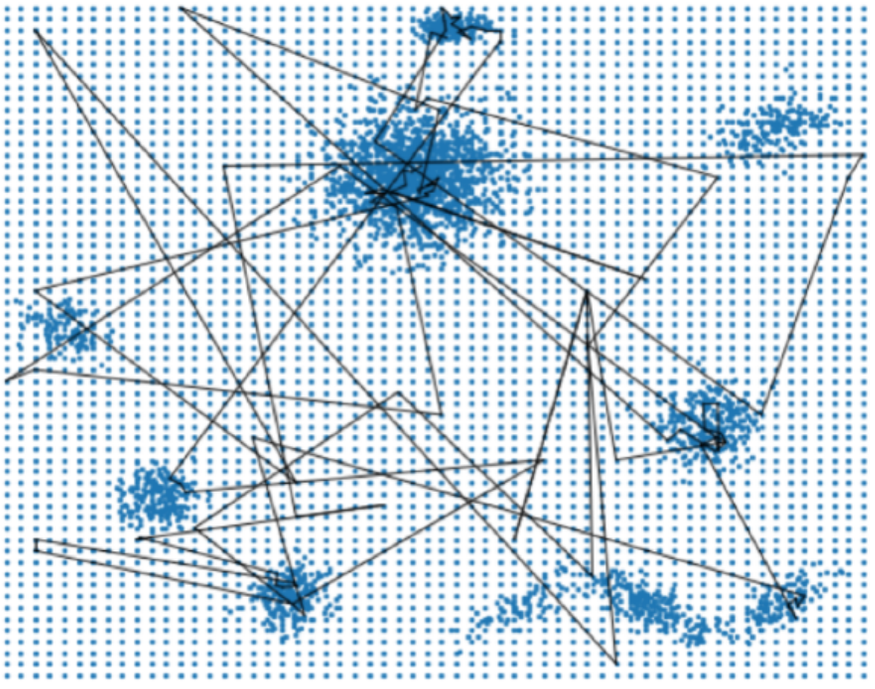
$\rightarrow 0.005 \rightarrow 0.005$

$\rightarrow 0.001 \rightarrow 0.001$

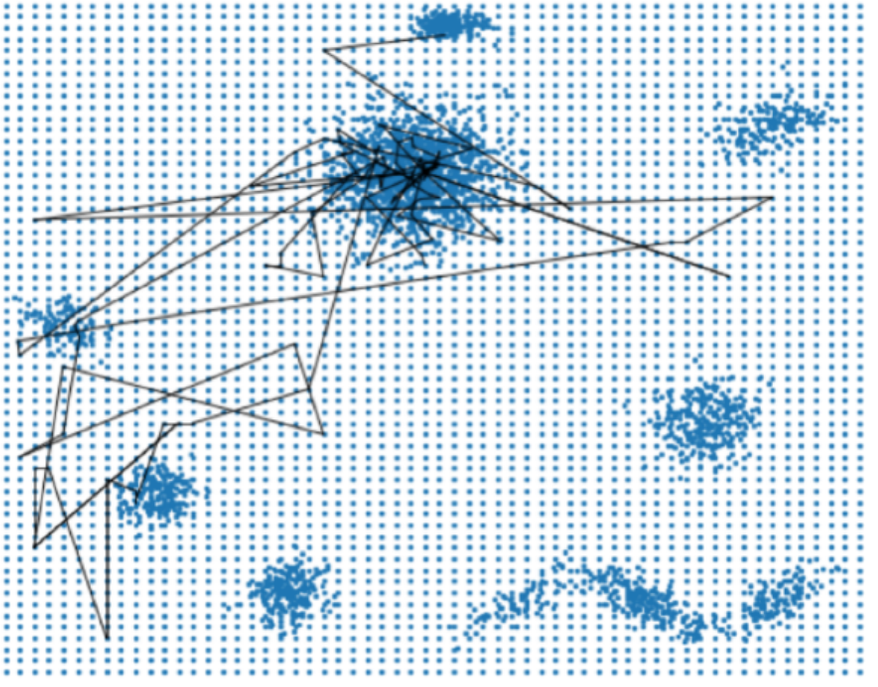
$\downarrow$ $\downarrow$ $\downarrow$

0.29 0.13 0.11

$\lambda_1 = 0.13$

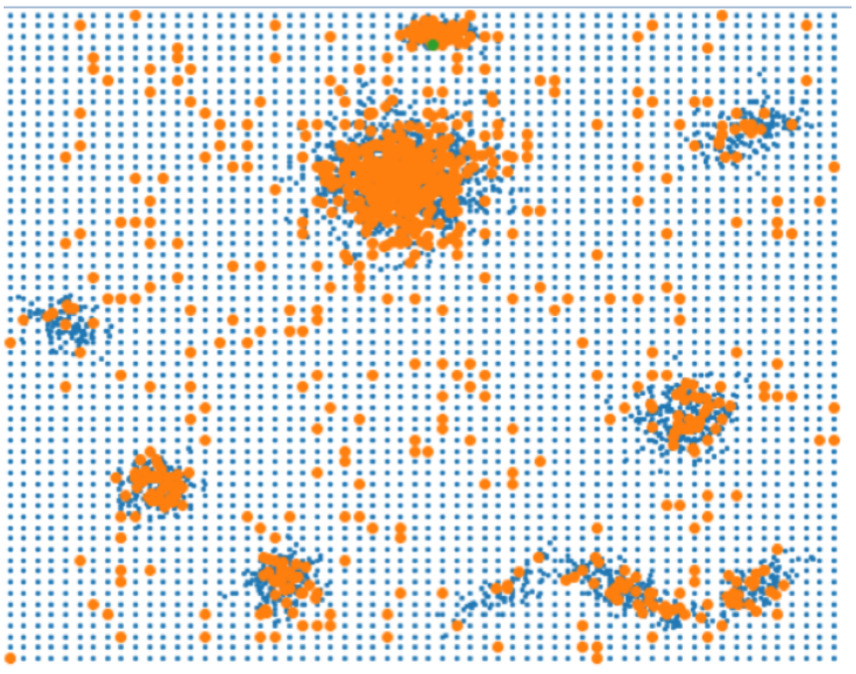


$\lambda_1 = 0.11$

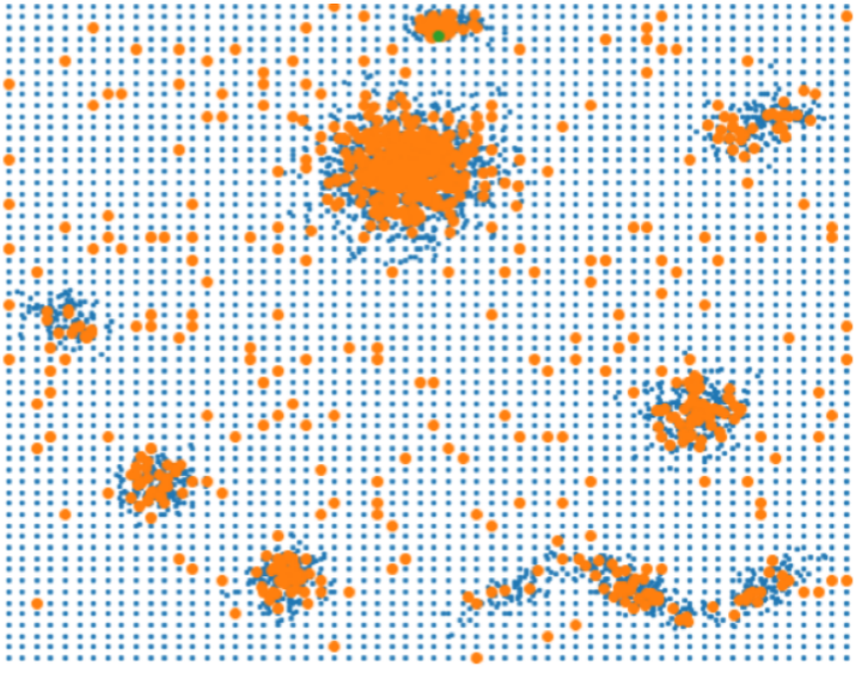


100 steps random walk

$\lambda_1 = 0.13$

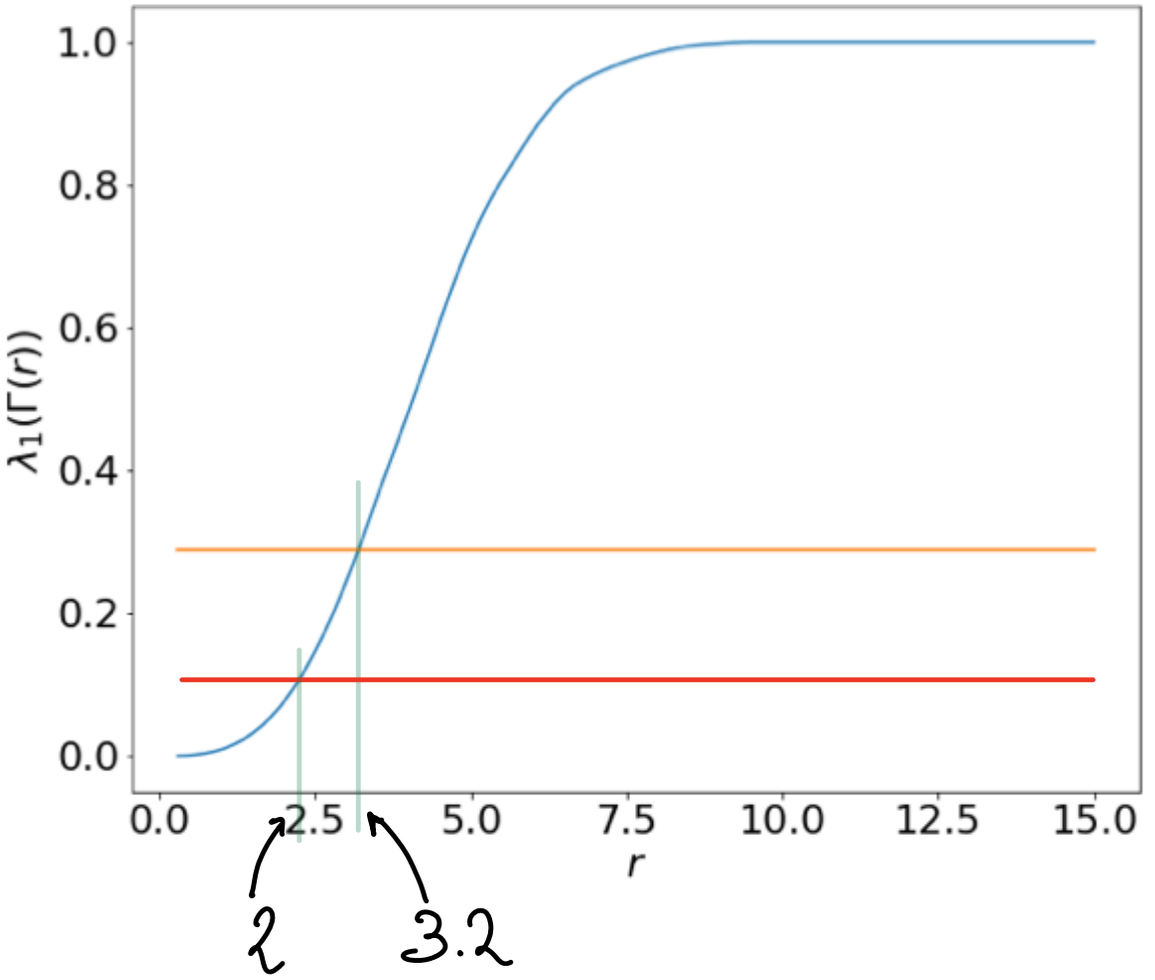


$\lambda_1 = 0.11$



sites visited with a walk replicated every 10 steps

$$\Gamma(r) : p(x,y) = \begin{cases} 1 & \text{if } d(x,y) \leq r \\ 0 & \text{otherwise} \end{cases}$$



λ_1 3,2 ≈ 386 km

λ_1' 2 ≈ 180 km