

Graphs with convex balls and groups acting on them

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Graphs with convex balls (called for short CB-graphs) were first studied separately in the eighties by Chepoi and Soltan and by Farber and Jamison. They generalize systolic and weakly systolic graphs, and enjoy some analogous properties to those of weakly modular and Helly graphs. However, unlike systolic graphs which were widely studied as they provide a combinatorial counterpart to CAT-0spaces, very few research has been done on CB-graphs.

We will see that CB-graphs enjoy some interesting properties which are known to be true in the other aforementioned classes of graphs. For example one can characterize them as those graphs satisfying two metric properties, called the Interval Neighborhood Condition and the Triangle Pentagon Condition. We will also see how to define clique paths in CB-graphs, which can be locally characterized. Thanks to a result of Swiatkowski, we will observe that it implies that groups acting geometrically on CB-graphs are biautomatic. Eventually we will see a local to global characterization of CB-graphs, giving a topological insight of their structure.

Joint work with Jérémie Chalopin and Victor Chepoi.