The diameter problem on graphs: a geometric point of view

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Computing the diameter of a graph (maximum distance between two of its vertices) is a fundamental problem with a well-known quadratic-time solution in the number of edges. During this talk, we will explore some connections between the existence of faster algorithms on special graph classes and their respective geometric properties. A well-studied such property in Metric Graph Theory is the existence of a distance-preserving embedding of a graph in a system or a product of trees. We will show that, if we are given an embedding in a constant number of trees, then we can compute all eccentricities in almost linear time. In doing so, we obtain the first almost linear-time algorithm for computing all eccentricities in various subclasses of partial cubes. For the median graphs, the most famous subclass of partial cubes, I will also briefly sketch the first subquadratic-time algorithm for computing their diameter. Interestingly, median graphs are also exactly the retracts of hypercubes. We will present additional results on the diameter problem both for the absolute retracts of irreflexive graphs (of which chordal bipartite graphs are a subclass) and the absolute retracts of reflexive graphs, a.k.a., Helly graphs. Finally, we will discuss about a recent framework for faster diameter computation, which is based on a VC-dimension argument and can be applied to all proper minor-closed graph classes. The latter framework allows us to completely characterize the hereditary subclasses of chordal graphs for which a subquadratic-time algorithm for diameter computation exists.