

# A De Bruijn – Erdős Theorem in Graphs?

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A set of  $n$  points in the Euclidean plane determines at least  $n$  distinct lines unless these  $n$  points are collinear. In 2006, Chen and Chvátal asked whether the same statement holds true in general metric spaces, where the line determined by points  $x$  and  $y$  is defined as the set consisting of  $x$ ,  $y$ , and all points  $z$  such that one of the three points  $x, y, z$  lies between the other two. The conjecture that it does hold true remains open even in the special case where the metric space is induced by a graph. There, a stronger conjecture has been suggested by computations carried out by Yori Zwols: In the set of all graphs on  $n$  non-collinear vertices, the number of distinct lines is minimized by a complete multipartite graph.