A Cartan-Hadamard theorem for median metric spaces

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A metric is said to be (globally) median, if any three points have a unique "median" which lies between any two points from the triple. Such spaces arise naturally in many different contexts. The property of being locally median can be viewed as a kind of non-positive curvature condition. We show that a complete uniformly locally median space is globally median if and only if it is simply connected. This is an analogue of the well known Cartan-Hadamard Theorem for non-positively curved manifolds, or more generally CAT(0) spaces. However it leaves open a number of interesting questions.