Numerical methods for highly-oscillatory evolution equations

Mohammed LEMOU
Institut de Mathématiques de Rennes, Université de Rennes 1

Abstract : Usual numerical methods become inefficient when they are applied to highly oscillatory evolution problems (order reduction or complete loss of accuracy). The numerical parameters must indeed be adapted to the high frequencies that come into play to correctly capture the desired information, and this induces a prohibitive computational cost. Furthermore, the numerical resolution of averaged models, even at high orders, is not sufficient to capture low frequencies and transition regimes. We present (very briefly) two strategies allowing to remove this obstacle for a large class of evolution problems : a 2-scale method and a micro/macro method. Two different frameworks will be considered : constant frequency, and variable - possibly vanishing - frequency. The result of these approaches is the construction of numerical schemes whose order of accuracy no longer depends on the frequency of oscillation, one then speaks of uniform accuracy (UA) for these schemes. Finally, a new technique for systematizing these two methods will be presented. Its purpose is to reduce the number of inputs that the user must provide to apply the method in practice. In other words, only the values of the field defining the evolution equation (and not its derivatives) are used. These methods have been successfully applied to solve a number of evolution models: non-linear Schrdinger and Klein-Gordon equations, Vlasov-Poisson kinetic equation with strong magnetic field, quantum transport in graphene.