

# The Landau equation: Particle Methods & Gradient Flow Structure

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**Abstract :** The Landau equation introduced by Landau in the 1930's is an important partial differential equation in kinetic theory. It gives a description of colliding particles in plasma physics, and it can be formally derived as a limit of the Boltzmann equation where grazing collisions are dominant. The purpose of this talk is to propose a new perspective inspired from gradient flows for weak solutions of the Landau equation, which is in analogy with the relationship of the heat equation and the 2-Wasserstein metric gradient flow of the Boltzmann entropy. Moreover, we aim at using this interpretation to derive a deterministic particle method to solve efficiently the Landau equation. Our deterministic particle scheme preserves all the conserved quantities at the semidiscrete level for the regularized Landau equation and that is entropy decreasing. We will illustrate the performance of these schemes with efficient computations using treecode approaches borrowed from multipole expansion methods for the 3D relevant Coulomb case. From the theoretical viewpoint, we use the theory of metric measure spaces for the Landau equation by introducing a bespoke Landau distance  $d_L$ . Moreover, we show for a regularized version of the Landau equation that we can construct gradient flow solutions, curves of maximal slope, via the corresponding variational scheme. The main result obtained for the Landau equation shows that the chain rule can be rigorously proved for the grazing continuity equation, this implies that H-solutions with certain a priori estimates on moments and entropy dissipation are equivalent to gradient flow solutions of the Landau equation. We crucially make use of estimates on Fisher information-like quantities in terms of the Landau entropy dissipation developed by Desvillettes.