A new stability and convergence proof of the Fourier-Galerkin spectral method for the spatially homogeneous Boltzmann equation

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Abstract : Numerical approximation of the Boltzmann equation is a challenging problem due to its high-dimensional, nonlocal, and nonlinear collision integral. Over the past decade, the Fourier-Galerkin spectral method has become a popular deterministic method for solving the Boltzmann equation, manifested by its high accuracy and potential of being further accelerated by the fast Fourier transform. Albeit its practical success, the stability of the method is only recently proved by Filbet, F. & Mouhot, C. in [Trans.Amer.Math.Soc. 363, no. 4 (2011): 1947-1980.] by utilizing the "spreading" property of the collision operator. In this work, we provide a new proof based on a careful L2 estimate of the negative part of the solution. We also discuss the applicability of the result to various initial data, including both continuous and discontinuous functions. This is joint work with Kunlun Qi and Tong Yang.