Modal based hypocoercivity methods on the torus and the real line with application to Goldstein-Taylor models

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Abstract : We are concerned with deriving sharp exponential decay estimates (i.e. with maximum rate and minimum multiplicative constant) for linear, hypocoercive evolution equations. Using a modal decomposition of the model allows to assemble a Lyapunov functional using Lyapunov matrix inequalities for each Fourier mode.

We shall illustrate the approach on the 1D Goldstein-Taylor model, a 2-velocity transport-relaxation equation. On the torus the lowest Fourier modes determine the spectral gap of the whole equation in L^2. By contrast, on the whole real line the Goldstein-Taylor model does not have a spectral gap, since the decay rate of the Fourier modes approaches zero in the small mode limit. Hence, the decay is reduced to algebraic.

In the final part of the talk we consider the Goldstein-Taylor model with non-constant relaxation rate, which is hence not amenable to a modal decomposition. In this case we construct a Lyapunov functional of pseudodifferential nature, one that is motivated by the modal analysis in the constant case. The robustness of this approach is illustrated on a multi-velocity Goldstein-Taylor model, yielding explicit rates of convergence to the equilibrium.

This is joint work with J. Dolbeault, A. Einav, C. Schmeiser, B. Signorello, and T. Wöhrer.

References

[1] A. Arnold, A. Einav, B. Signorello, T. Wöhrer: Large-time convergence of the non-homogeneous Goldstein-Taylor equation, J. Stat. Phys. 182 (2021) 41.

[2] A. Arnold, J. Dolbeault, C. Schmeiser, T. Wöhrer: Sharpening of decay rates in Fourier based hypocoercivity methods, To appear in INdAM proceedings (2021).