

CHROMATIC HOMOTOPY, K -THEORY AND FUNCTORS

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TUESDAY 15:30 – 16:30 , **Yonatan Harpaz** (CNRS–Université Paris 13):

The universal property of topological Hochschild homology.

Topological Hochschild homology is a fundamental invariant of rings and ring spectra, related to algebraic K -theory via the celebrated Dennis-Bökstedt trace map $K \rightarrow THH$. Blumberg, Gepner and Tabuada showed that algebraic K -theory becomes especially well-behaved when considered as an invariant of stable ∞ -categories, rather than just ring spectra: in that setting it can be described as the free additive invariant generated by the unit, that is, the initial additive functor under the the core ∞ -groupoid functor, corepresented by the unit of Cat^{ex} . In this talk I will describe joint work with Thomas Nikolaus and Victor Saunier showing that THH similarly acquires a universal property when extended to stable ∞ -categories, when one allows in addition to take coefficients in an arbitrary bimodule. In particular, we view THH as a functor on the category TCat^{ex} whose objects are pairs (C, M) where C is a stable ∞ -category and M is a bimodule, that is, a biexact functor $C^{\text{op}} \times C \rightarrow \text{Spectra}$. We define a notion of being a trace-like invariant on TCat^{ex} , which amounts to sending certain maps in TCat^{ex} to equivalences. We then show that THH is the free exact trace-like invariant generated from the unit of TCat^{ex} , where exact means exact in the bimodule entry. At the same time, algebraic K -theory can also be extended to TCat^{ex} , in the form of endomorphism K -theory. Comparing universal properties we then get that THH is universally obtained from endomorphism K -theory by forcing exactness. This yields a conceptual proof that THH is the first Goodwillie derivative of endomorphism K -theory, and can be used to extend the Dundas-Goodwillie-McCarthy theorem to the setting of stable ∞ -categories.