Combinatory Categorial Grammars as Acceptors of Weighted Tree Languages

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Joint Work with Andreas Maletti

Weighted Automata: Theory and Applications

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Why is CCG interesting?

- widely used in **computational linguistics**

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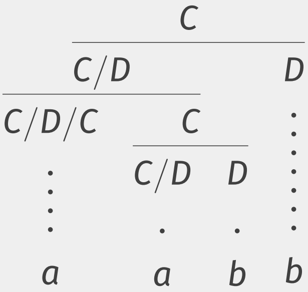
- widely used in **computational linguistics**
- **mildly context-sensitive** grammar formalism
- **efficiently parsable**
- **constituency-based** structures

EXAMPLE: $a^n b^n$

- input alphabet $\Sigma = \{a, b\}$
- atomic categories $A = \{C, D\}$
- initial categories $I = \{C\}$
- lexicon L with
  $L(a) = \{C/D,\ C/D/C\}$
  $L(b) = \{D\}$



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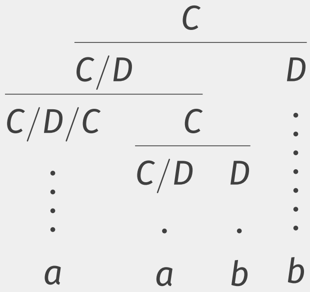
$$\frac{C\alpha}{C\alpha/D \quad D} \qquad\qquad \frac{C\alpha}{C\alpha/C \quad C}$$



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$$\frac{C\alpha/D \quad D}{C\alpha} \qquad \frac{C\alpha/C \quad C}{C\alpha}$$

$$\frac{C\alpha/C \quad C/D}{C\alpha/D} \qquad \frac{C\alpha/C \quad C/D/C}{C\alpha/D/C}$$

$$\dfrac{\dfrac{\overset{\textstyle C/D/C}{\vdots \atop a} \quad \dfrac{\overset{\textstyle C/D}{\vdots \atop a} \quad \overset{\textstyle D}{\vdots \atop b}}{C}}{C/D} \quad \overset{\textstyle D}{\vdots \atop b}}{C}$$

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$$\frac{C\alpha/C \quad C/D}{C\alpha/D} \qquad \frac{C\alpha/C \quad C/D/C}{C\alpha/D/C}$$

$$\frac{D/D\backslash C \quad D\alpha\backslash D}{D\alpha/D\backslash C}$$

$$\cfrac{\cfrac{\underset{a}{\underset{\vdots}{C/D/C}} \quad \cfrac{\underset{a}{\underset{\vdots}{C/D}} \quad \underset{b}{\underset{\vdots}{D}}}{C}}{C/D} \quad \underset{b}{\underset{\vdots}{D}}}{C}$$

Adding Weights

$$\frac{C\alpha}{C\alpha/D \quad D} \qquad \frac{C\alpha}{C\alpha/C \quad C} \qquad \frac{C\alpha/D}{C\alpha/C \quad C/D} \qquad \frac{C\alpha/D/C}{C\alpha/C \quad C/D/C}$$

$$\dfrac{\dfrac{C/D/C \qquad \dfrac{C/D \quad D}{C}}{C/D} \qquad D}{C}$$

$$\frac{C\alpha}{C\alpha/D \quad D} \qquad \frac{C\alpha}{C\alpha/C \quad C} \qquad \frac{C\alpha/D}{C\alpha/C \quad C/D} \qquad \frac{C\alpha/D/C}{C\alpha/C \quad C/D/C}$$

$$\dfrac{\dfrac{C/D/C \qquad \dfrac{C/D \quad D}{C}}{C/D} \qquad D}{C} \qquad\qquad \dfrac{\dfrac{\dfrac{C/D/C \quad C/D}{C/D/D} \qquad D}{C/D} \qquad D}{C}$$

$$\frac{C\alpha}{C\alpha/D \quad D}\ 1 \qquad \frac{C\alpha}{C\alpha/C \quad C}\ 1 \qquad \frac{C\alpha/D}{C\alpha/C \quad C/D}\ 2 \qquad \frac{C\alpha/D/C}{C\alpha/C \quad C/D/C}\ 2$$

$$\frac{\dfrac{C/D/C \qquad \dfrac{C/D \quad D}{C}}{C/D} \qquad D}{C} \qquad\qquad \frac{\dfrac{\dfrac{C/D/C \quad C/D}{C/D/D} \qquad D}{C/D} \qquad D}{C}$$

Adding Weights

$$\frac{C\alpha}{C\alpha/D \quad D}\;1 \qquad \frac{C\alpha}{C\alpha/C \quad C}\;1 \qquad \frac{C\alpha/D}{C\alpha/C \quad C/D}\;2 \qquad \frac{C\alpha/D/C}{C\alpha/C \quad C/D/C}\;2$$

$$\dfrac{\dfrac{C/D/C \qquad \dfrac{C/D \quad D}{C}}{C/D} \qquad D}{C} \qquad\qquad \dfrac{\dfrac{\dfrac{C/D/C \quad C/D}{C/D/D} \qquad D}{C/D} \qquad D}{C}$$

$$1 \cdot 1 \cdot 1 = 1$$

$$\frac{C\alpha}{C\alpha/D \quad D}\ 1 \qquad \frac{C\alpha}{C\alpha/C \quad C}\ 1 \qquad \frac{C\alpha/D}{C\alpha/C \quad C/D}\ 2 \qquad \frac{C\alpha/D/C}{C\alpha/C \quad C/D/C}\ 2$$

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$$1 \cdot 1 \cdot 1 = 1 \qquad\qquad 2 \cdot 1 \cdot 1 = 2$$

Relabeling

$$\frac{C\alpha}{C\alpha/D \quad D}\ 1 \qquad \frac{C\alpha}{C\alpha/C \quad C}\ 1 \qquad \frac{E\alpha}{E\alpha/F \quad F}\ 2 \qquad \frac{E\alpha}{E\alpha/E \quad E}\ 2$$

$$\dfrac{\dfrac{C/D/C \qquad \dfrac{C}{C/D \quad D}}{C/D} \qquad D}{C} \qquad\qquad \dfrac{\dfrac{E/F/E \qquad \dfrac{E}{E/F \quad F}}{E/F} \qquad F}{E}$$

$$1 \cdot 1 \cdot 1 = 1 \qquad\qquad 2 \cdot 2 \cdot 2 = 8$$

Relabeling

$$\frac{C\alpha}{C\alpha/D \quad D}\;1 \qquad \frac{C\alpha}{C\alpha/C \quad C}\;1 \qquad \frac{E\alpha}{E\alpha/F \quad F}\;2 \qquad \frac{E\alpha}{E\alpha/E \quad E}\;2$$

$$\dfrac{\dfrac{C/D/C \qquad \dfrac{C/D \quad D}{C}}{C/D} \qquad D}{C} \qquad \dfrac{\dfrac{E/F/E \qquad \dfrac{E/F \quad F}{E}}{E/F} \qquad F}{E} \quad \overset{\rho}{\longrightarrow} \quad \dfrac{\dfrac{a \qquad \dfrac{a \quad a}{a}}{a} \qquad a}{a}$$

$$1 \cdot 1 \cdot 1 = 1 \qquad\qquad 2 \cdot 2 \cdot 2 = 8$$

$$\frac{C\alpha}{C\alpha/D \quad D}\;1 \qquad \frac{C\alpha}{C\alpha/C \quad C}\;1 \qquad \frac{E\alpha}{E\alpha/F \quad F}\;2 \qquad \frac{E\alpha}{E\alpha/E \quad E}\;2$$

$$\frac{\dfrac{C/D/C \quad \dfrac{C/D \quad D}{C}}{C/D} \qquad D}{C} \qquad \frac{\dfrac{E/F/E \quad \dfrac{E/F \quad F}{E}}{E/F} \qquad F}{E} \quad \xrightarrow{\rho} \quad \frac{\dfrac{a \quad \dfrac{a \quad a}{a}}{a} \qquad a}{a}$$

$$1 \cdot 1 \cdot 1 = 1 \qquad\qquad 2 \cdot 2 \cdot 2 = 8 \qquad\qquad 1 + 8 = 9$$

Relabeling

$$\frac{C\alpha}{C\alpha/{\color{red}D} \quad {\color{red}D}}\ 1 \qquad \frac{C\alpha}{C\alpha/{\color{red}C} \quad {\color{red}C}}\ 1 \qquad \frac{E\alpha}{E\alpha/{\color{red}F} \quad {\color{red}F}}\ 2 \qquad \frac{E\alpha}{E\alpha/{\color{red}E} \quad {\color{red}E}}\ 2$$

$$\frac{\dfrac{C/D/C \quad \dfrac{C/D \quad D}{C}}{C/D} \quad D}{C} \qquad \frac{\dfrac{E/F/E \quad \dfrac{E/F \quad F}{E}}{E/F} \quad F}{E} \quad \xrightarrow{\rho} \quad \frac{\dfrac{a \quad \dfrac{a \quad a}{a}}{a} \quad a}{a}$$

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commutative semiring $(S, +, \cdot, 0, 1)$

Tree Language Expressive Power

0-CCG	1-CCG	k-CCG

[1]Kuhlmann, Maletti, S: The Tree-Generative Capacity of CCG (2019)
[2]S, Maletti: Strong Equivalence of TAG and CCG (to appear)

0-CCG	1-CCG	*k*-CCG
$\subsetneq$ RTG[1]		

RTG Regular Tree Grammar

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Tree Language Expressive Power

0-CCG	1-CCG	k-CCG
$\subsetneq$ RTG[1]	$=$ RTG[1]	$=$ sCFTG[1,2]

RTG Regular Tree Grammar
sCFTG Simple Monadic Context-Free Tree Grammar

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Weighted Tree Language Expressive Power

0-wCCG	1-wCCG	k-wCCG

Weighted Tree Language Expressive Power

0-wCCG	1-wCCG	k-wCCG
$\subsetneq$ wRTG		

wRTG Weighted Regular Tree Grammar

Weighted Tree Language Expressive Power

0-wCCG	1-wCCG	k-wCCG
$\subsetneq$ wRTG	$=$ wRTG	

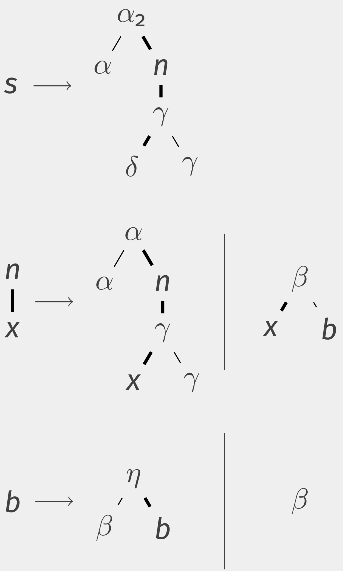
wRTG Weighted Regular Tree Grammar

Weighted Tree Language Expressive Power

0-wCCG	1-wCCG	k-wCCG
$\subsetneq$ wRTG	$=$ wRTG	$\subseteq$ wsCFTG

wRTG Weighted Regular Tree Grammar
wsCFTG Weighted Simple Monadic Context-Free Tree Grammar

Simple Monadic Context-Free Tree Grammar



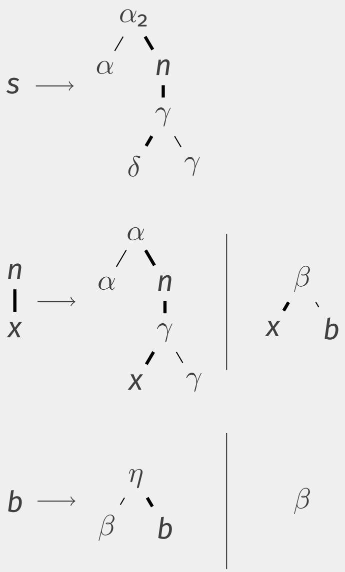
$G = (N, \Sigma, S, P)$ with

$N_0 = \{s, b\}, N_1 = \{n\}, S = \{s\},$

$\Sigma_0 = \{\alpha, \beta, \gamma, \delta\},$

$\Sigma_2 = \{\alpha, \beta, \gamma, \eta\}$

s



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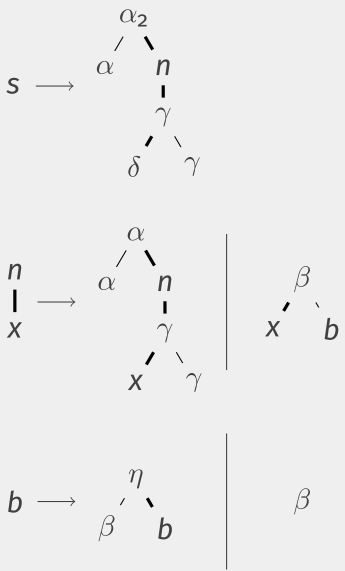

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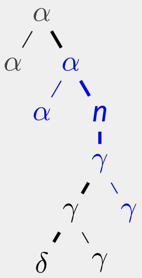

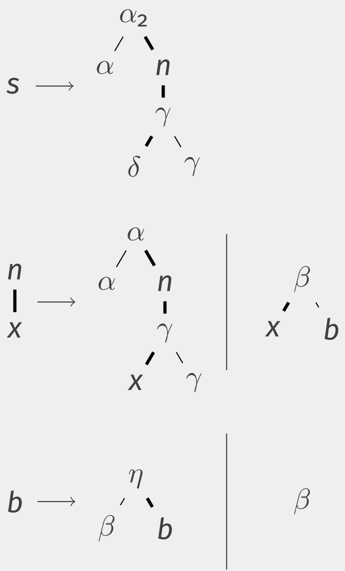

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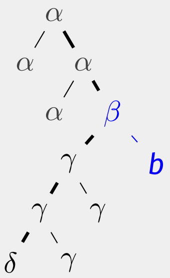

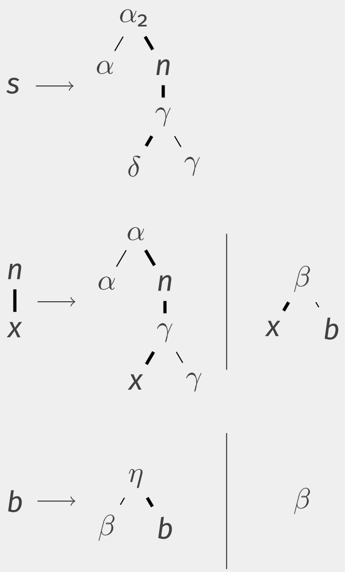

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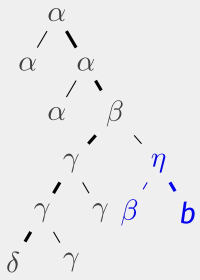

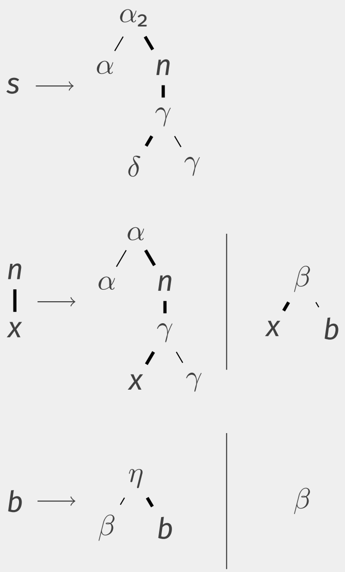

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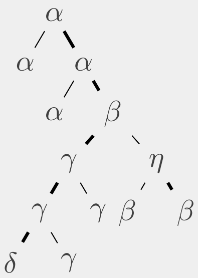
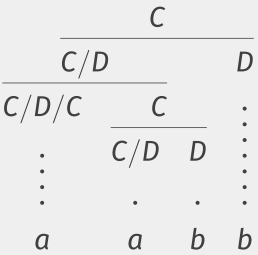
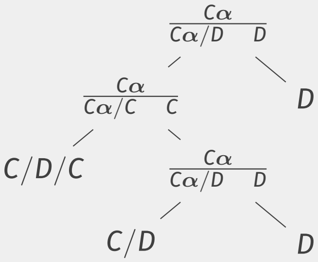


k-CCG $\subseteq$ sCFTG: Rule Trees

- there can be an **unlimited number of categories** in the CCG derivation trees



- regard **rule trees** as a natural encoding



k-CCG $\subseteq$ sCFTG: Nonterminals

- **nullary nonterminals**: categories with at most d arguments

$$N_0 = \{\langle C\rangle, \langle D\rangle, \langle C/C\rangle, \langle C/D\rangle, \\ \dots, \langle D\backslash D\backslash C\rangle, \langle D\backslash D\backslash D\rangle\}$$

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- **unary nonterminals**: rule placeholders of form
⟨ target, argument , argument context ⟩

$$N_1 = \{\langle C, \backslash C, \square\rangle, \quad \ldots, \quad \langle C, /D, \backslash C/D\rangle\}$$

$$\frac{C\alpha}{C \quad C\alpha\backslash C} \qquad\qquad \frac{C\alpha\backslash C/D}{C\alpha/D \quad D\backslash C/D}$$

k-CCG $\subseteq$ sCFTG: Example Derivation

$\langle C \rangle$

k-CCG $\subseteq$ sCFTG: Example Derivation

$$\langle C \rangle \Rightarrow \begin{array}{c} \langle C, /D, \Box \rangle \\ | \\ \langle C/D \rangle \end{array}$$

k-CCG $\subseteq$ sCFTG: Example Derivation

$$\langle C \rangle \Rightarrow \begin{array}{c} \langle C, /D, \square \rangle \\ | \\ \langle C/D \rangle \end{array}$$

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$$\langle C \rangle \Rightarrow \begin{array}{c} \langle C, /D, \square \rangle \\ | \\ \langle C/D \rangle \end{array} \Rightarrow \begin{array}{c} \langle C, /D, \square \rangle \\ | \\ \langle C, /C, \square \rangle \\ | \\ \langle C/D/C \rangle \end{array}$$

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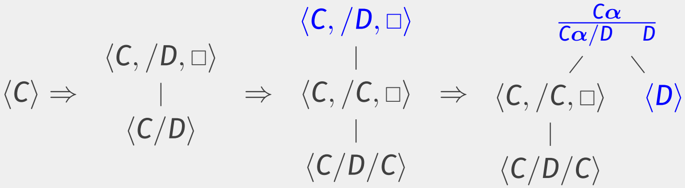

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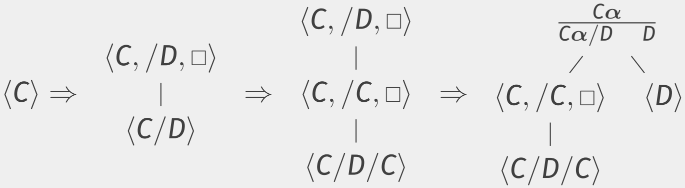

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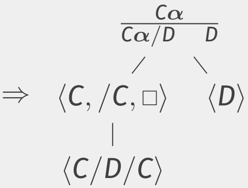
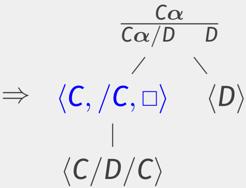


k-CCG $\subseteq$ sCFTG: Example Derivation



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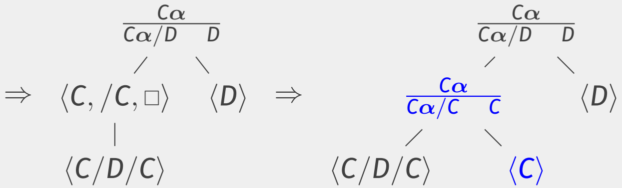
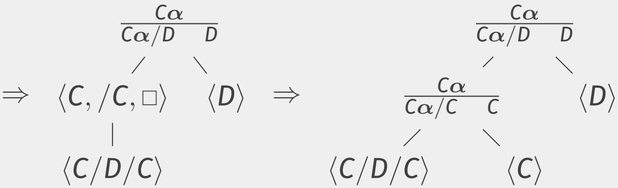


k-CCG ⊆ sCFTG: Example Derivation



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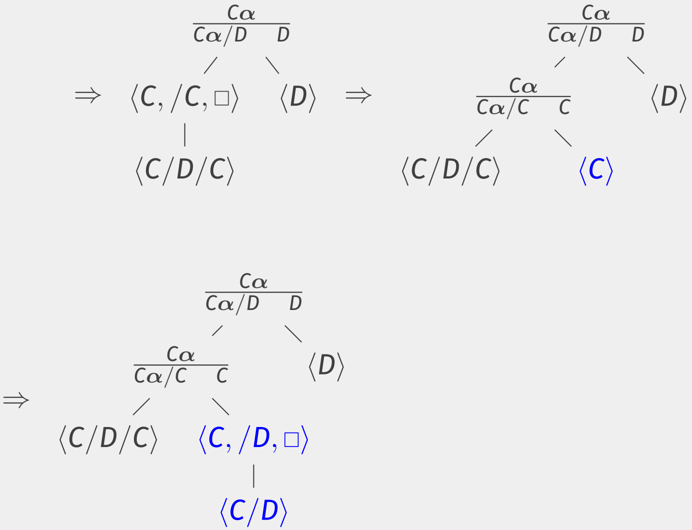

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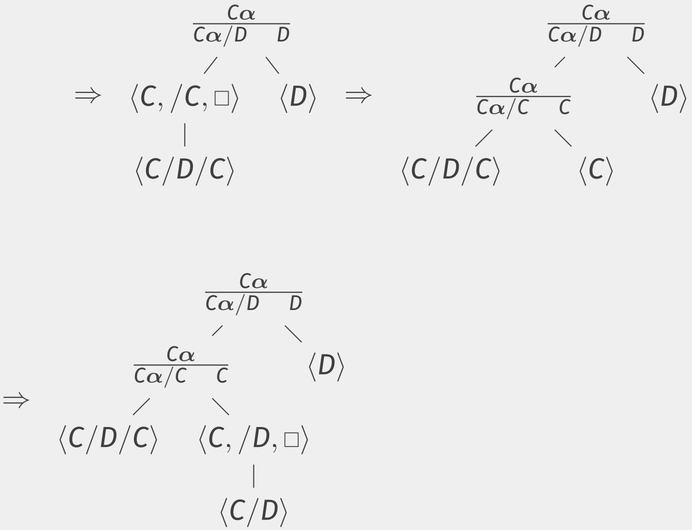
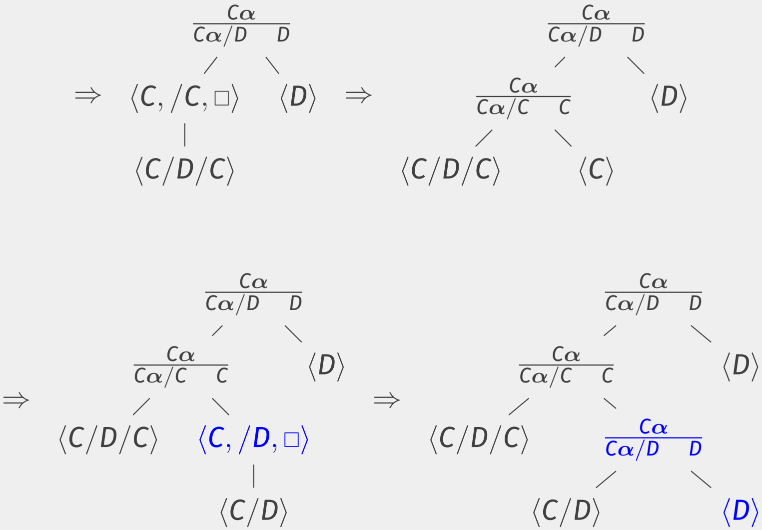
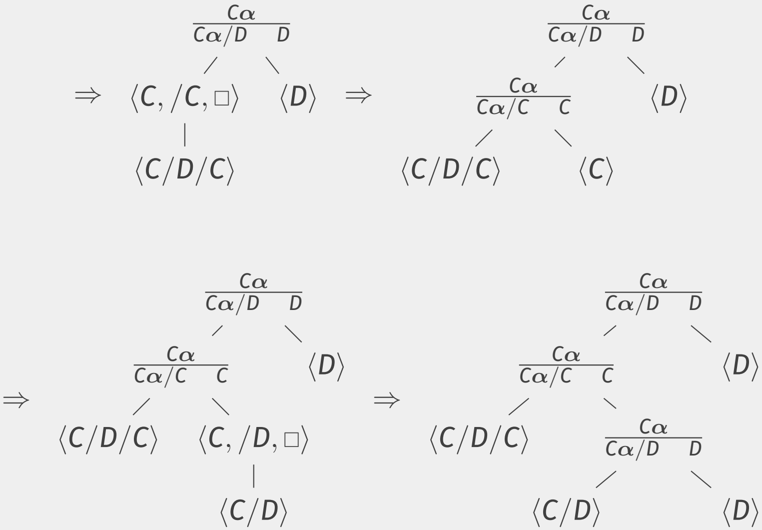


k-CCG $\subseteq$ sCFTG: Example Derivation

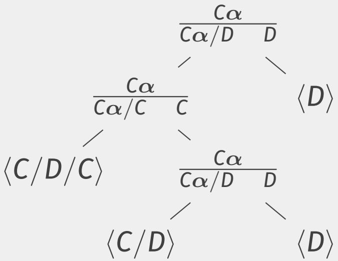


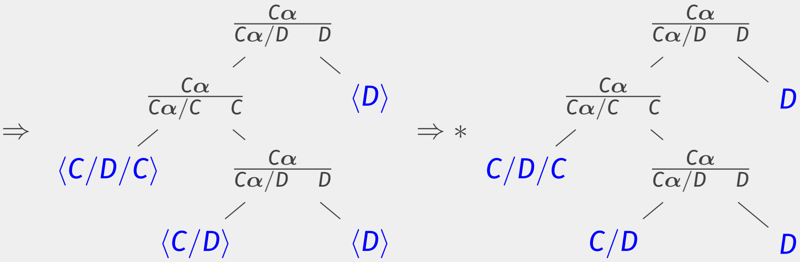
k-CCG ⊆ sCFTG: Example Derivation



k-CCG $\subseteq$ sCFTG: Example Derivation

$\Rightarrow$

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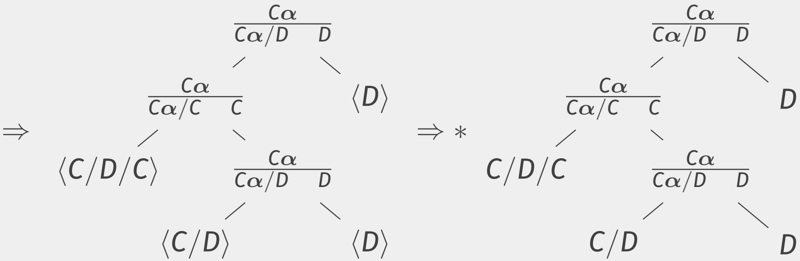
k-CCG $\subseteq$ sCFTG: Example Derivation



- **expand nullary nonterminals**

$$\langle C \rangle \longrightarrow \begin{array}{c} \langle C, /D, \Box \rangle \\ | \\ \langle C/D \rangle \end{array}$$

k-CCG $\subseteq$ sCFTG: Productions

- **expand nullary nonterminals**

$$\langle C \rangle \quad \longrightarrow \quad \begin{array}{c} \langle C, /D, \square \rangle \\ | \\ \langle C/D \rangle \end{array}$$

- **expand unary nonterminals**

$$\begin{array}{c} \langle C, /D, \square \rangle \\ | \\ x \end{array} \longrightarrow \begin{array}{c} \langle C, \backslash C, \square \rangle \\ | \\ \langle C, /D, \backslash C \rangle \\ | \\ x \end{array}$$

k-CCG ⊆ sCFTG: Productions

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$$\langle C \rangle \longrightarrow \begin{array}{c} \langle C, /D, \Box \rangle \\ | \\ \langle C/D \rangle \end{array}$$

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- **produce nullary terminal symbols**

$$\langle C/D \rangle \to C/D$$
$$\langle C/D/C \rangle \to C/D/C$$
$$\langle D \rangle \to D$$

- **expand nullary nonterminals**

$$\langle C\rangle \longrightarrow \begin{array}{c}\langle C, /D, \square\rangle \\ \mid \\ \langle C/D\rangle\end{array}$$

- **expand unary nonterminals**

$$\begin{array}{c}\langle C, /D, \square\rangle \\ \mid \\ x\end{array} \longrightarrow \begin{array}{c}\langle C, \backslash C, \square\rangle \\ \mid \\ \langle C, /D, \backslash C\rangle \\ \mid \\ x\end{array}$$

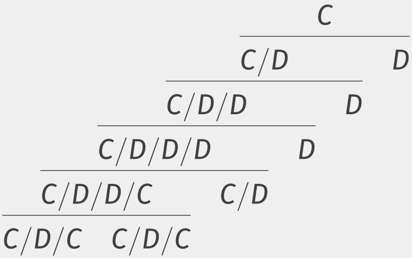
- **produce nullary terminal symbols**

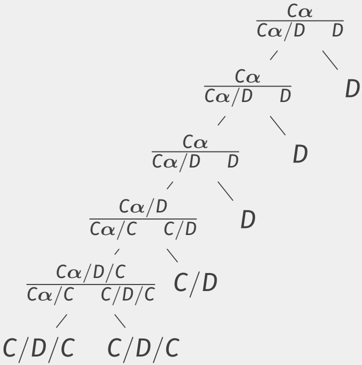
$$\langle C/D\rangle \to C/D$$
$$\langle C/D/C\rangle \to C/D/C$$
$$\langle D\rangle \to D$$

- **produce binary terminal symbols**

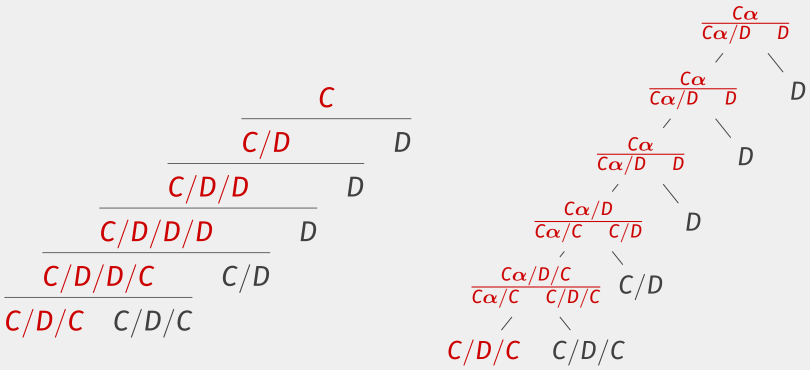
$$\begin{array}{c}\langle C, /D, \square\rangle \\ \mid \\ x\end{array} \longrightarrow \begin{array}{c}\dfrac{C\alpha}{C\alpha/D \quad D} \\ / \quad \backslash \\ x \quad \langle D\rangle\end{array}$$

k-CCG $\subseteq$ sCFTG: Eliminating Ambiguity





k-CCG $\subseteq$ sCFTG: Eliminating Ambiguity



k-CCG ⊆ sCFTG: Eliminating Ambiguity

C

C/D

$C/D/D$

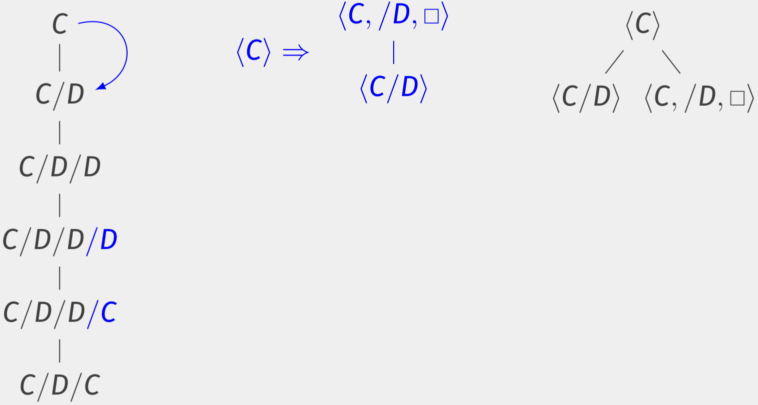
$C/D/D/D$

$C/D/D/C$

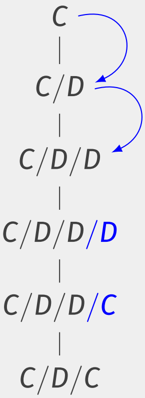
$C/D/C$

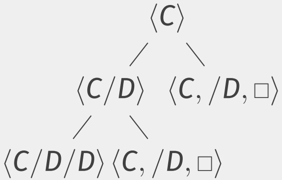
$\langle C \rangle$

k-CCG $\subseteq$ sCFTG: Eliminating Ambiguity



k-CCG ⊆ sCFTG: Eliminating Ambiguity





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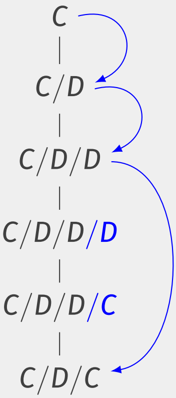

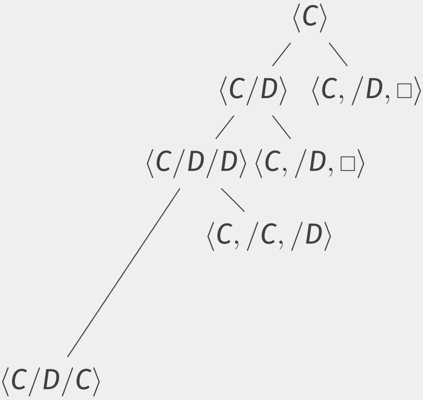

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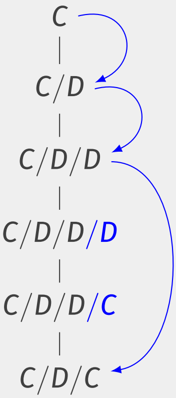
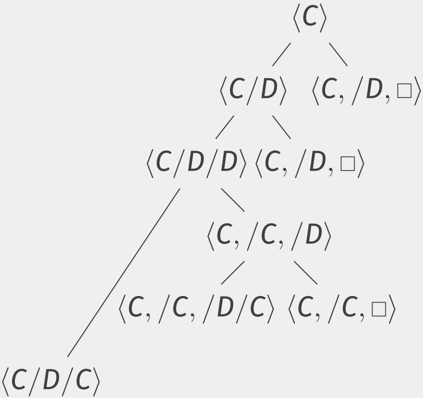
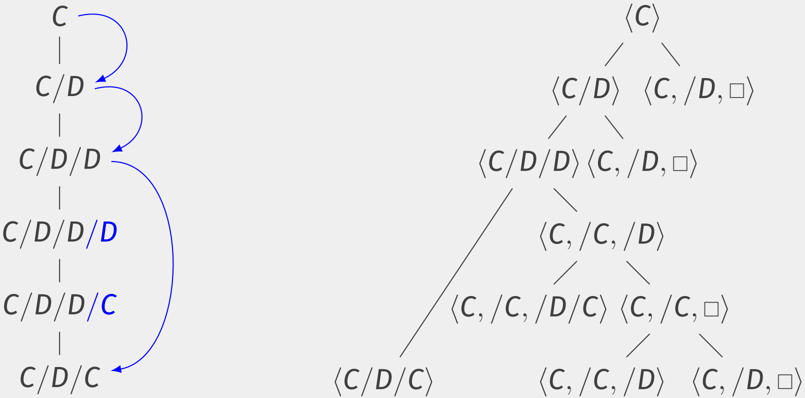
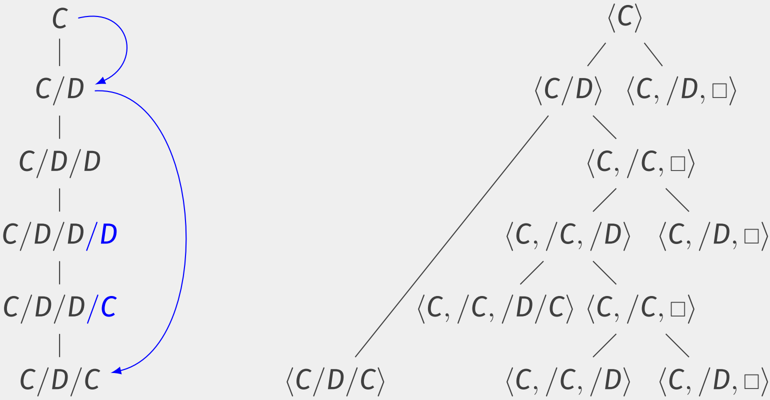




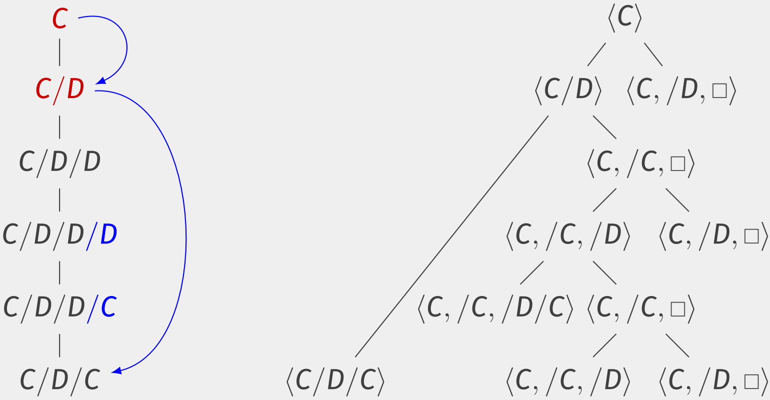
k-CCG $\subseteq$ sCFTG: Eliminating Ambiguity



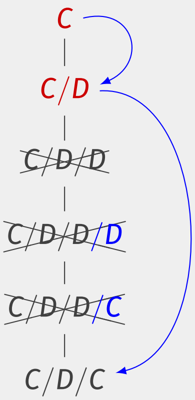
k-CCG $\subseteq$ sCFTG: Eliminating Ambiguity

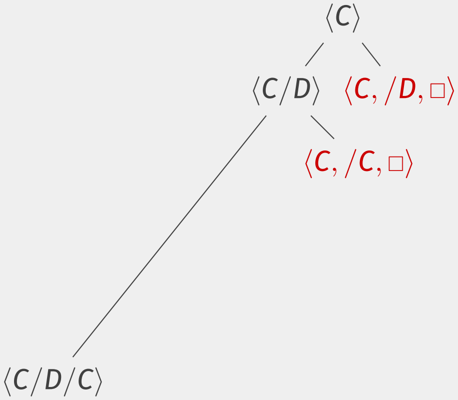


k-CCG $\subseteq$ sCFTG: Eliminating Ambiguity



k-CCG $\subseteq$ sCFTG: Eliminating Ambiguity





0-wCCG	1-wCCG	k-wCCG
$\subsetneq$ wRTG	$=$ wRTG	$\subseteq$ wsCFTG

wRTG Weighted Regular Tree Grammar

wsCFTG Weighted Simple Monadic Context-Free Tree Grammar

0-wCCG	1-wCCG	k-wCCG
$\subsetneq$ wRTG	$=$ wRTG	$\subseteq$ wsCFTG

wRTG Weighted Regular Tree Grammar

wsCFTG Weighted Simple Monadic Context-Free Tree Grammar

Thank you for your attention!