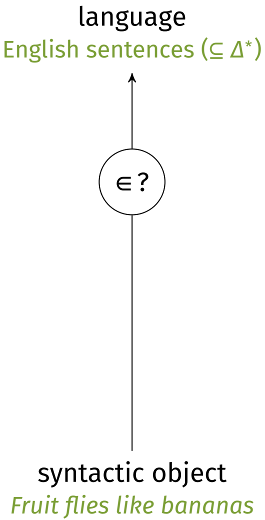
Weighted Parsing for Grammar-Based Language Models over Multioperator Monoids

WATA 2020/2021

Richard Mörbitz   Heiko Vogler

2021-04-23

Weighted parsing problems



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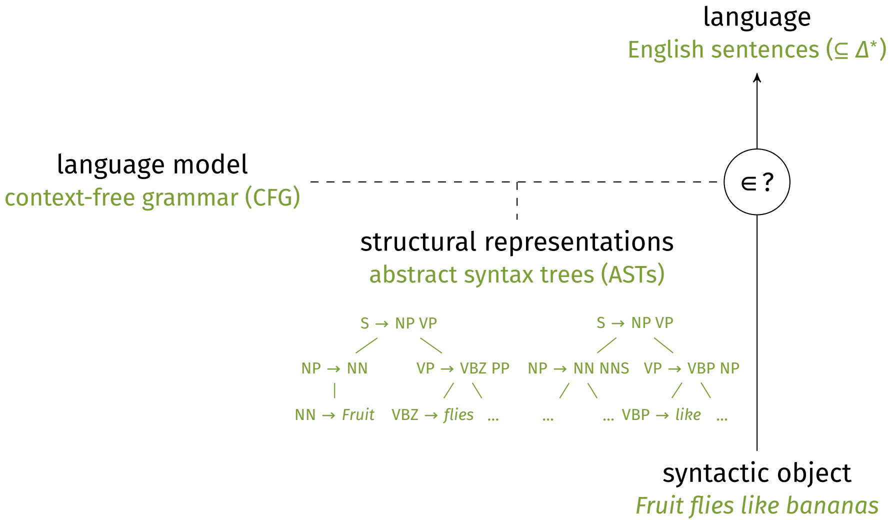

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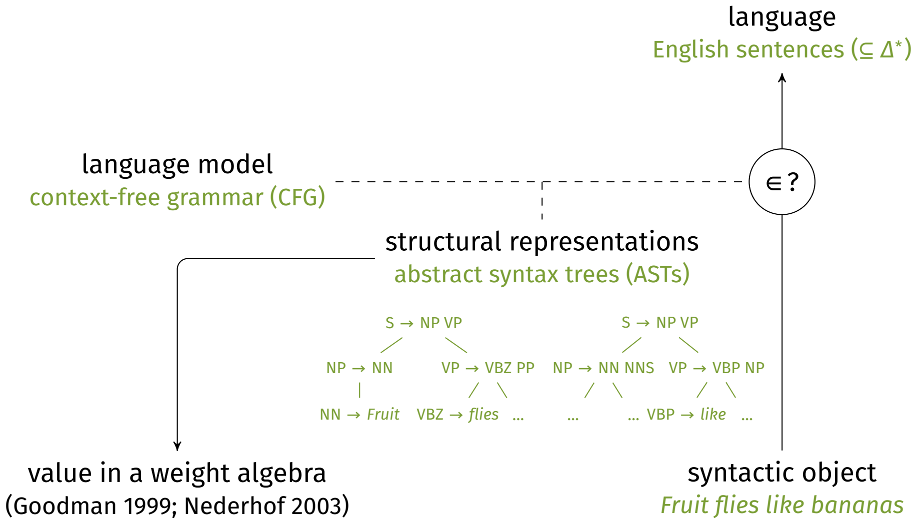

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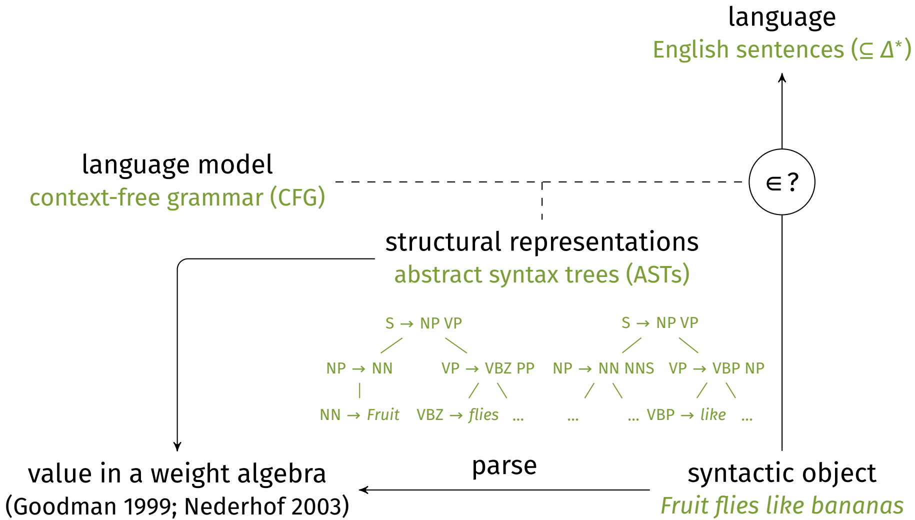

Outline

Our formalism

Weighted RTG-based language model

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Regular tree grammar

$G = (N, \Sigma, A_0, R)$

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$(\mathcal{L}, \phi)$ is a Σ-algebra

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- Multioperator monoid
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from semiring $(\mathbb{K}, \oplus, \odot, \mathbb{0}, \mathbb{1})$
with $\Omega = \{\odot^m_{\Bbbk} \mid m \in \mathbb{N}, \Bbbk \in \mathbb{K}\}$

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wt

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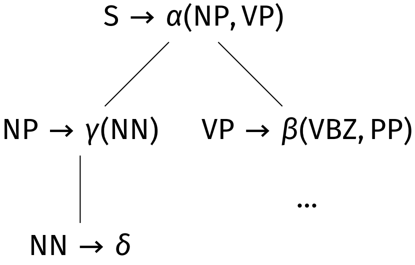
Weighted *RTG-based* language models

Tuple $G = (N, \Sigma, A_0, R)$

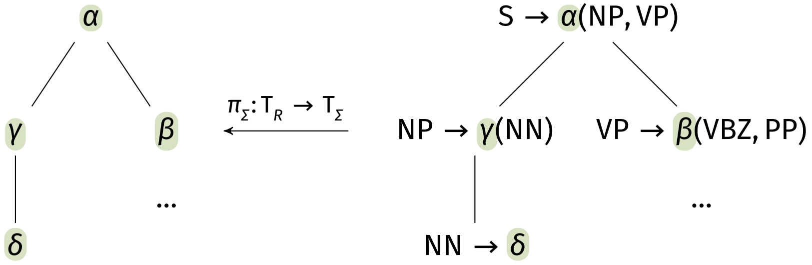
Example rules:

$$\begin{aligned} \mathrm{S} &\to \alpha(\mathrm{NP}, \mathrm{VP}) \\ \mathrm{VP} &\to \beta(\mathrm{VBZ}, \mathrm{PP}) \\ \mathrm{NP} &\to \gamma(\mathrm{NN}) \\ \mathrm{NN} &\to \delta \\ &\dots \end{aligned}$$



abstract syntax tree $d \in \mathrm{AST}(G)$

Weighted RTG-based *language models*



$t \in L(G) \subseteq \mathrm{T}_\Sigma$

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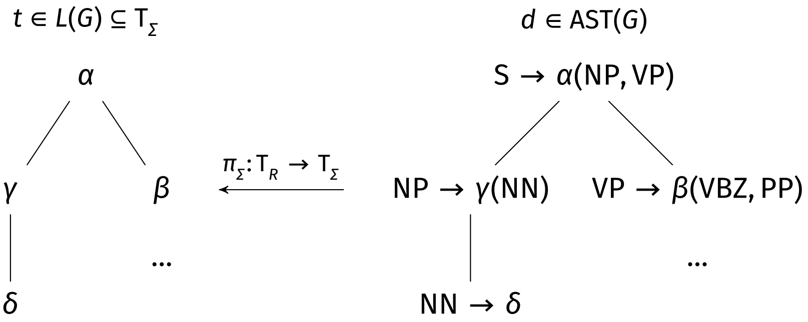

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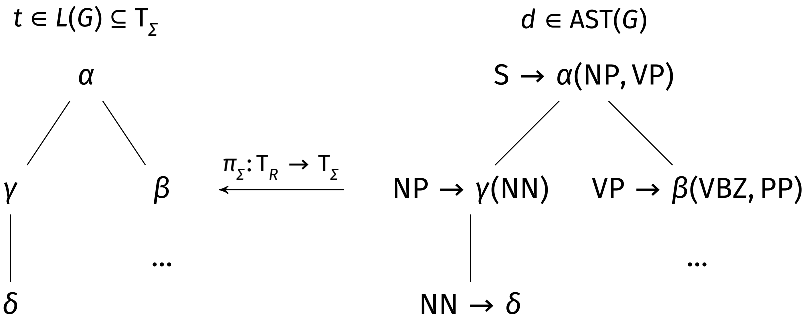

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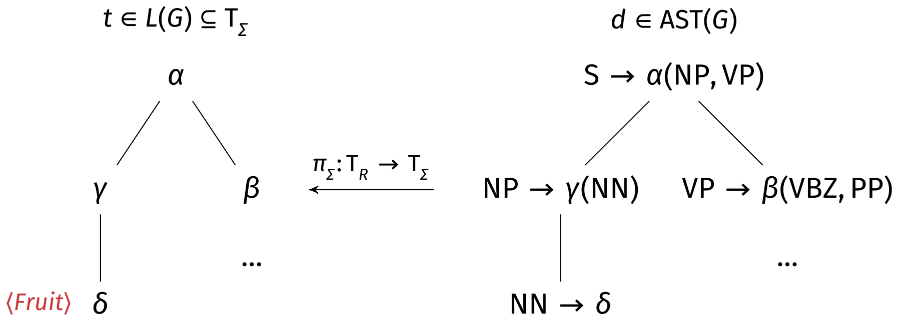

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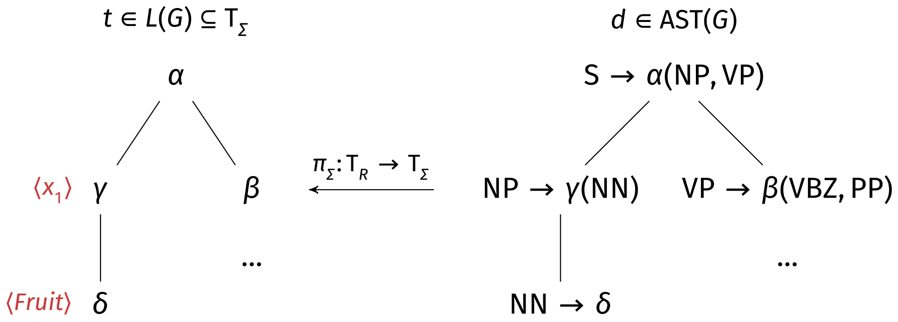

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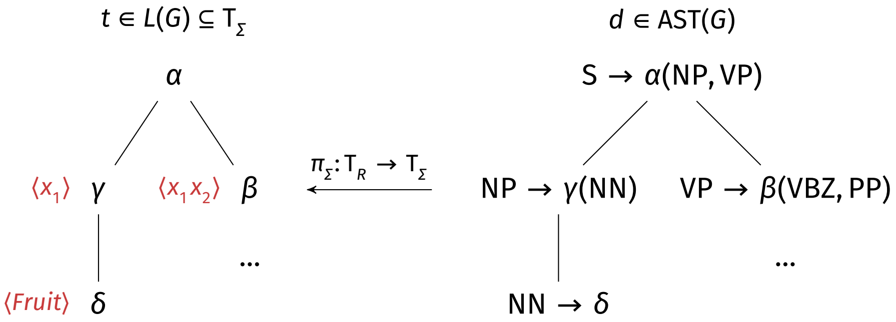

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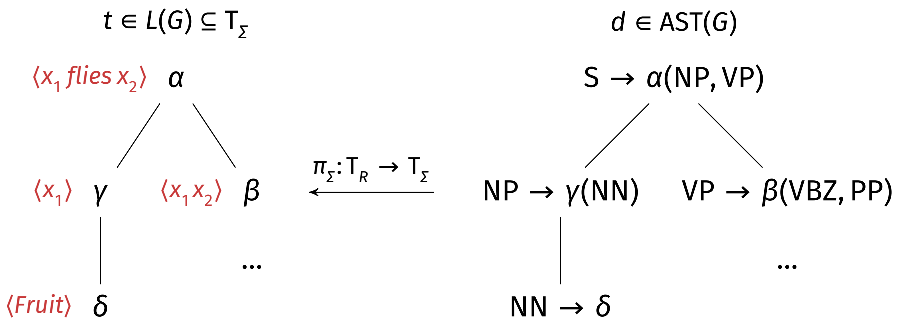

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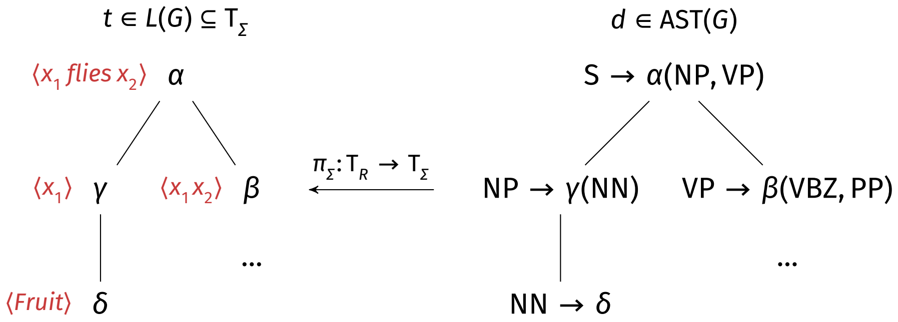

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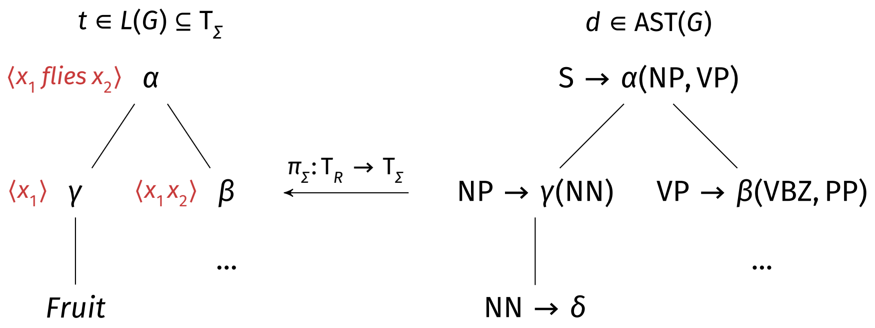

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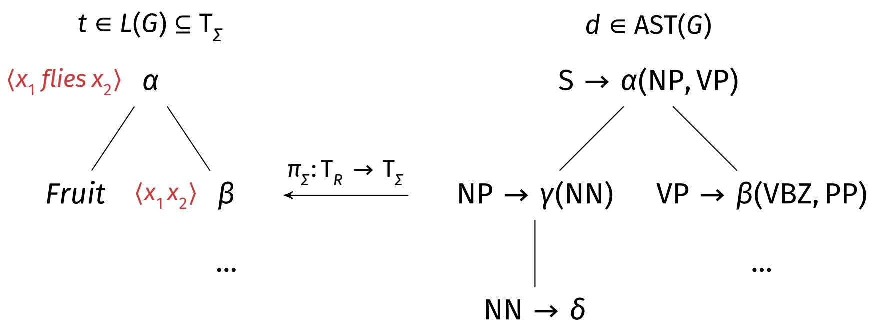

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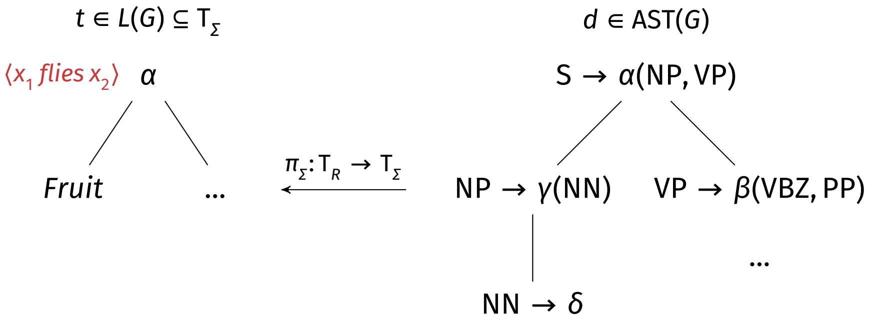

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Weighted RTG-based *language models*

$t \in L(G) \subseteq \mathrm{T}_\Sigma$

$\langle \textit{Fruit flies} \ldots \rangle$

$d \in \mathrm{AST}(G)$

$\mathrm{S} \to \alpha(\mathrm{NP}, \mathrm{VP})$

$\pi_\Sigma : \mathrm{T}_R \to \mathrm{T}_\Sigma$ ←

$\mathrm{NP} \to \gamma(\mathrm{NN})$ $\quad$ $\mathrm{VP} \to \beta(\mathrm{VBZ}, \mathrm{PP})$

$\mathrm{NN} \to \delta$ $\quad$ …

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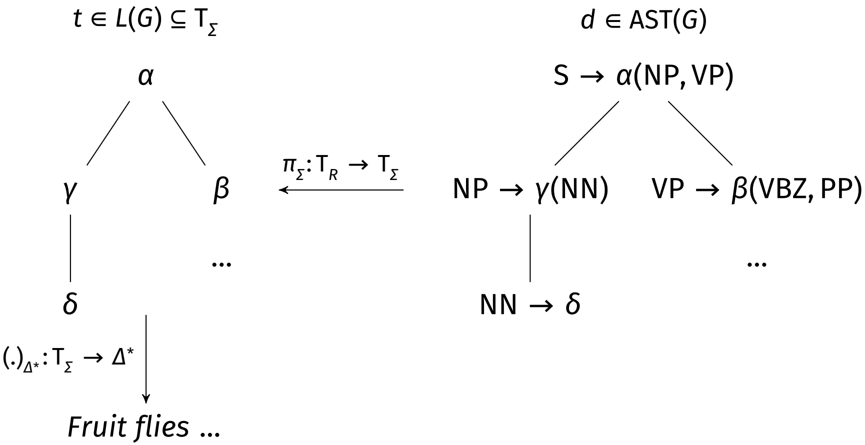

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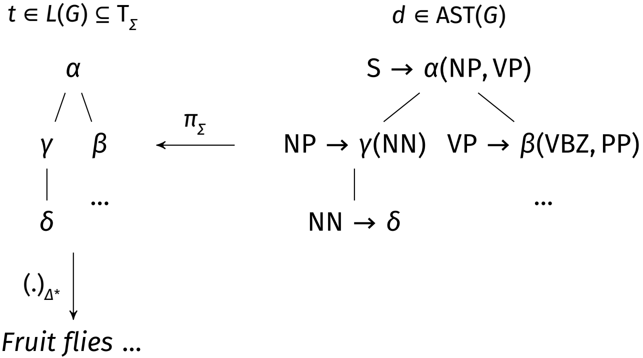

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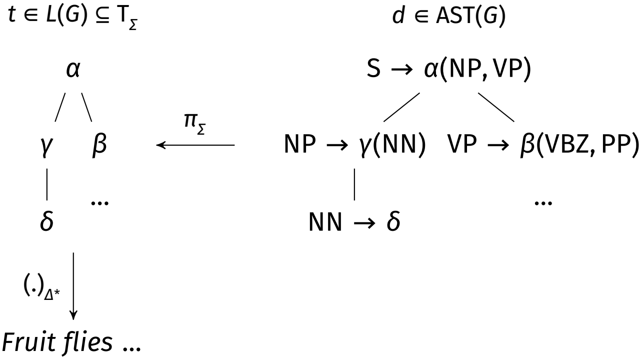

- wt: R (set of rules) $\to \Omega$ (set of operations) $(\mathbb{R}_0^1, \max, 0, \Omega_{\mathrm{mul}})$

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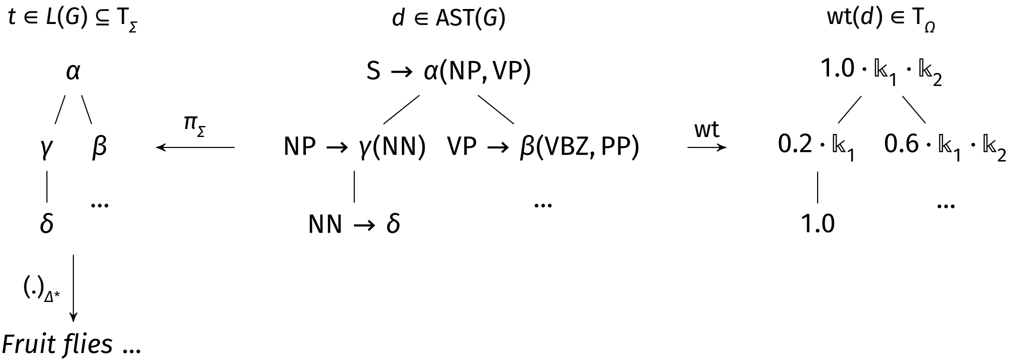

- wt: R (set of rules) $\to$ Ω (set of operations)

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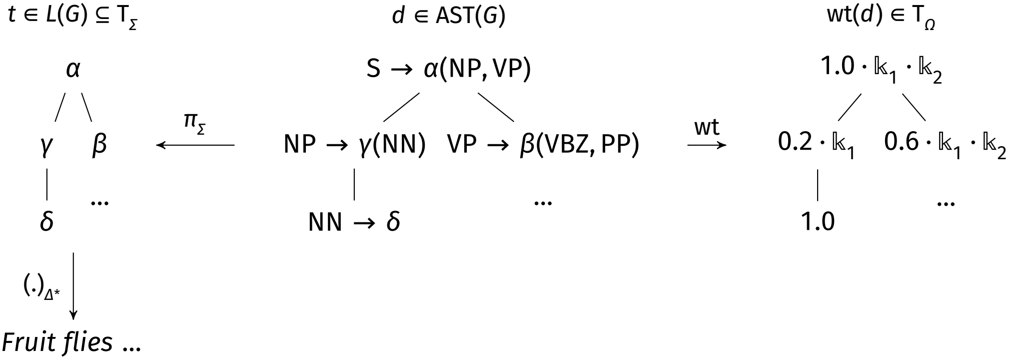

- wt: R (set of rules) $\to$ Ω (set of operations)
- $(.)_{\mathbb{R}_1^0}$: T_Ω (terms) $\to$ $\mathbb{R}_0^1$ (weight algebra)

$(\mathbb{R}_0^1, \max, 0, \Omega_{\mathrm{mul}})$

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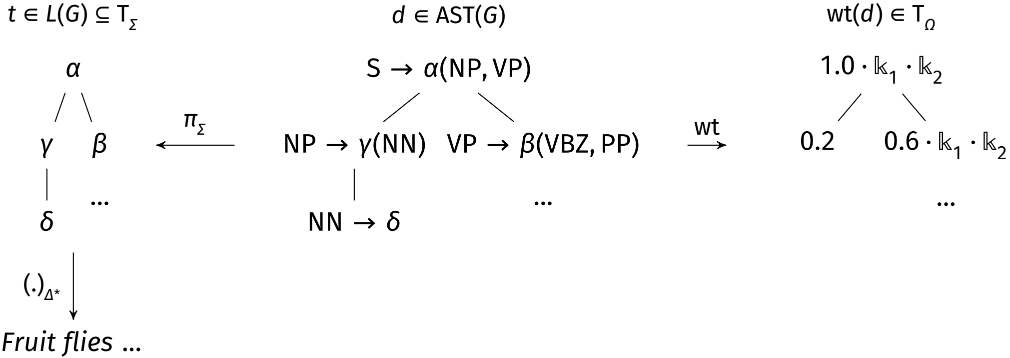

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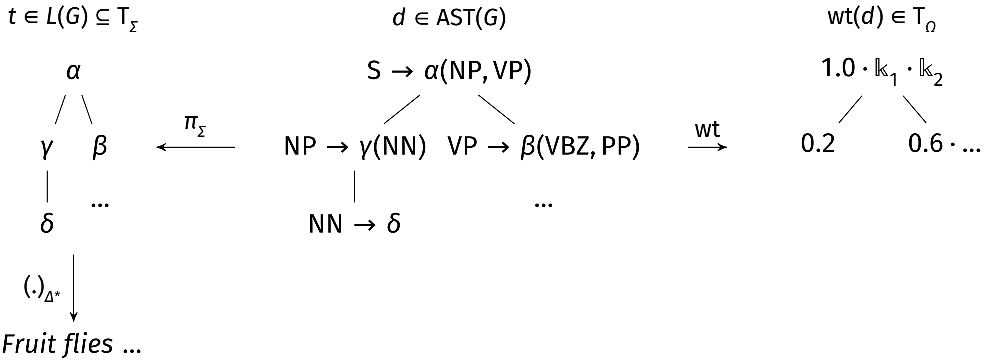

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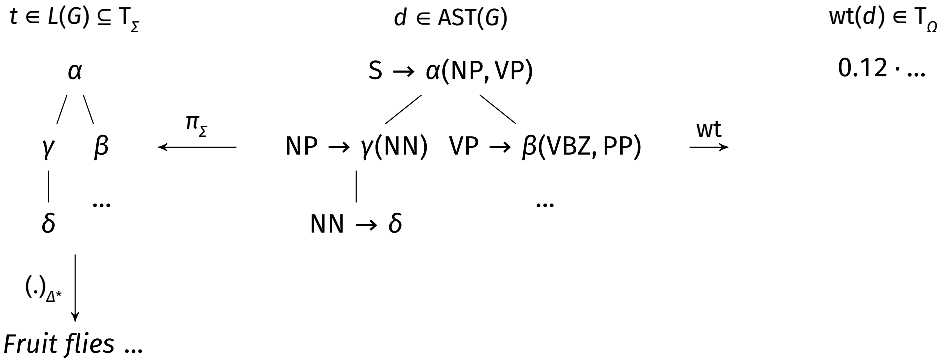

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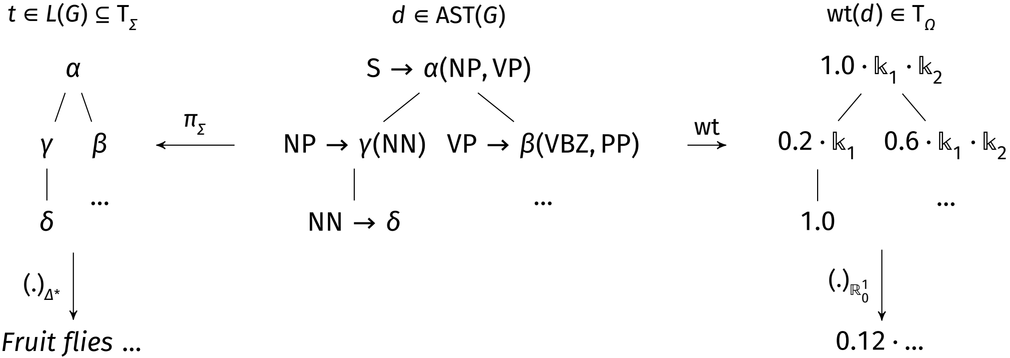

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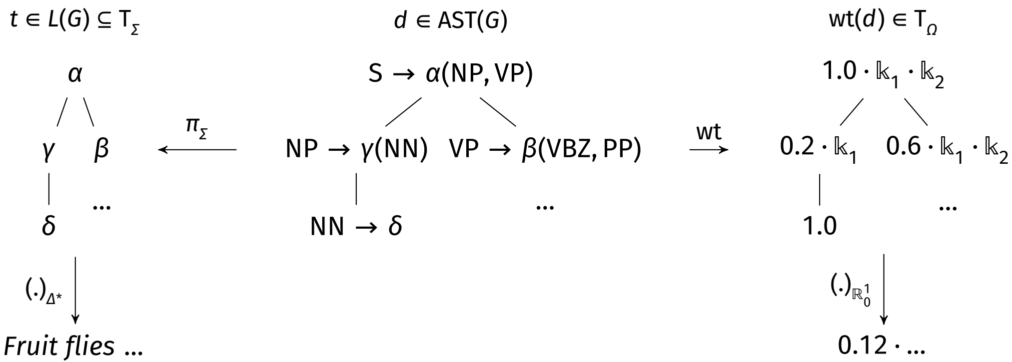

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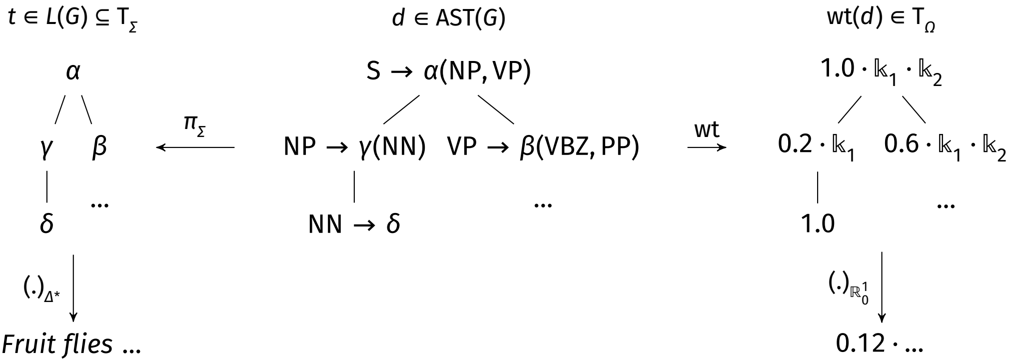
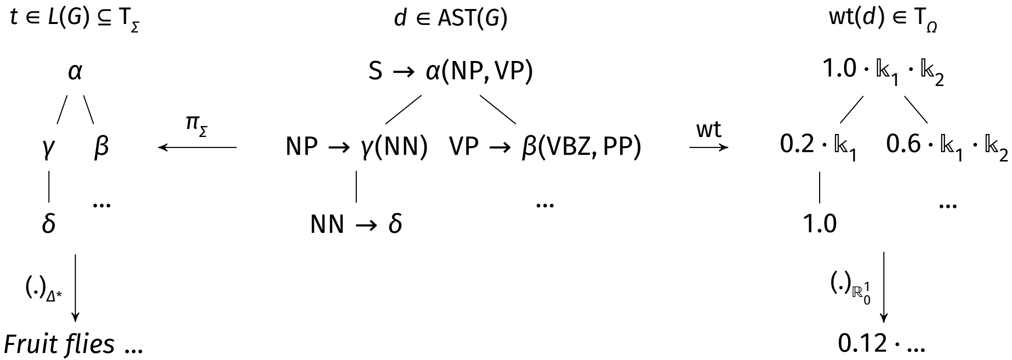


Definition (weighted RTG-based language model)

A wRTG-LM is a tuple $\Big(\underbrace{(G = (N, \Sigma, A_0, R))}_{\text{RTG}}, \underbrace{(\mathcal{L}, \phi)}_{\text{language algebra}}), \underbrace{(\mathbb{K}, \oplus, \mathbb{0}, \Omega)}_{\text{M-monoid}}, \underbrace{\mathrm{wt}}_{\mathrm{wt}\colon R \rightarrow \Omega}\Big)$.

Outline

The M-monoid parsing problem



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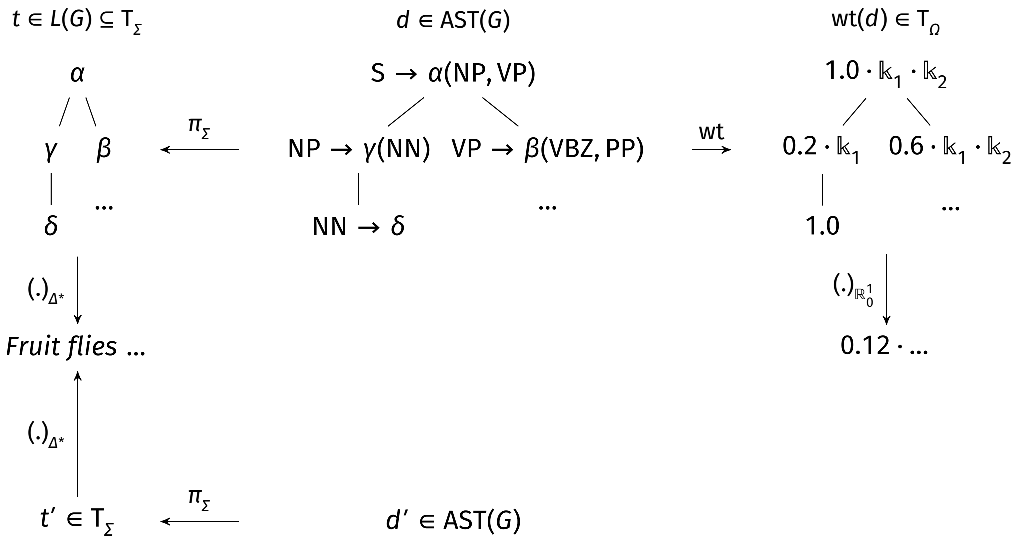

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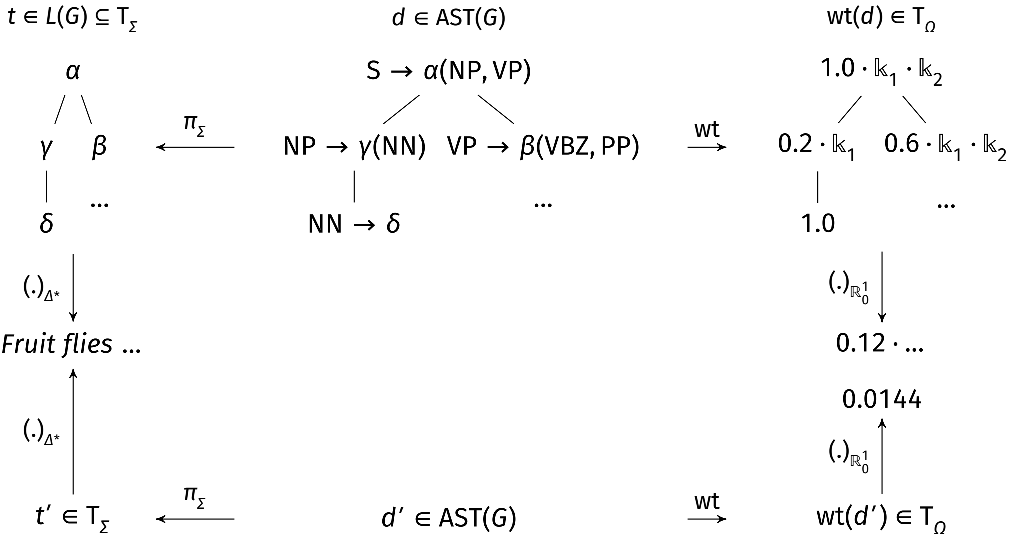

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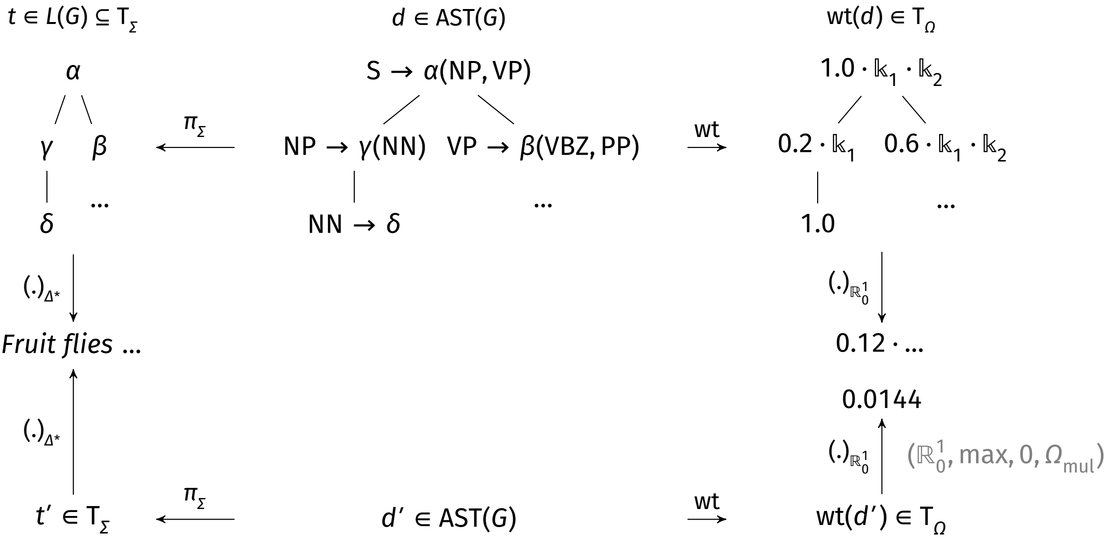

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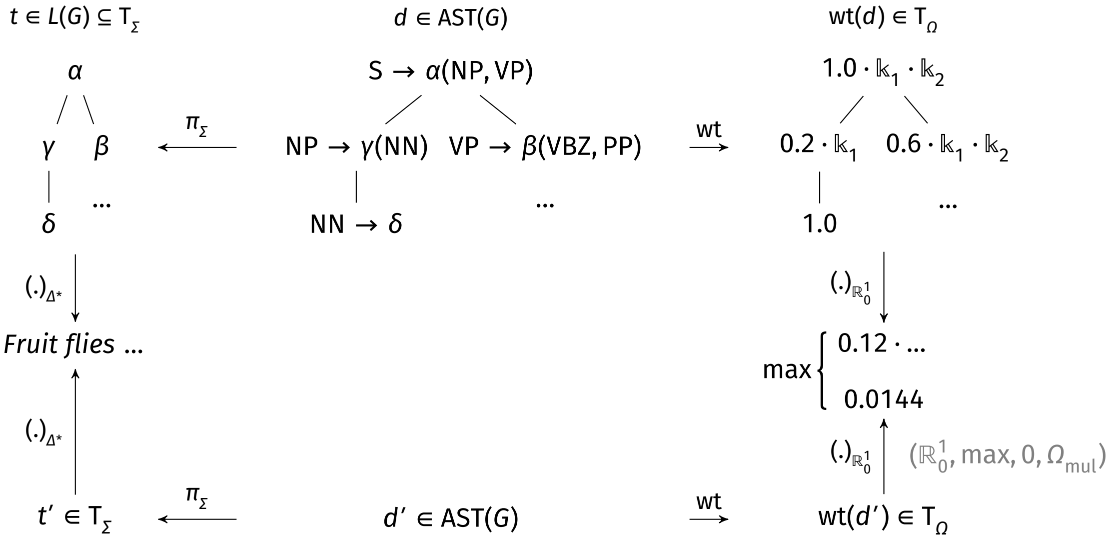

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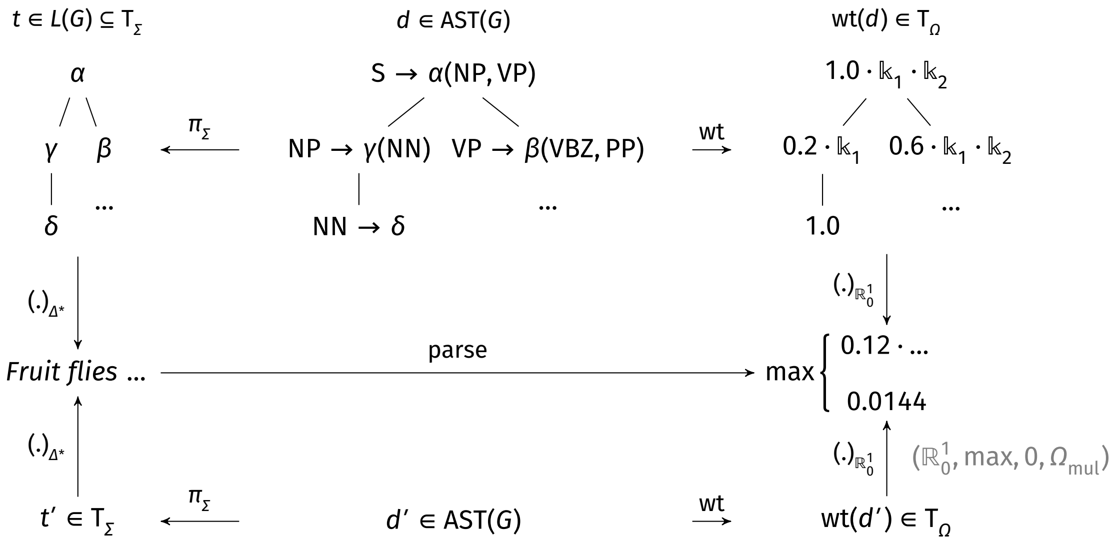

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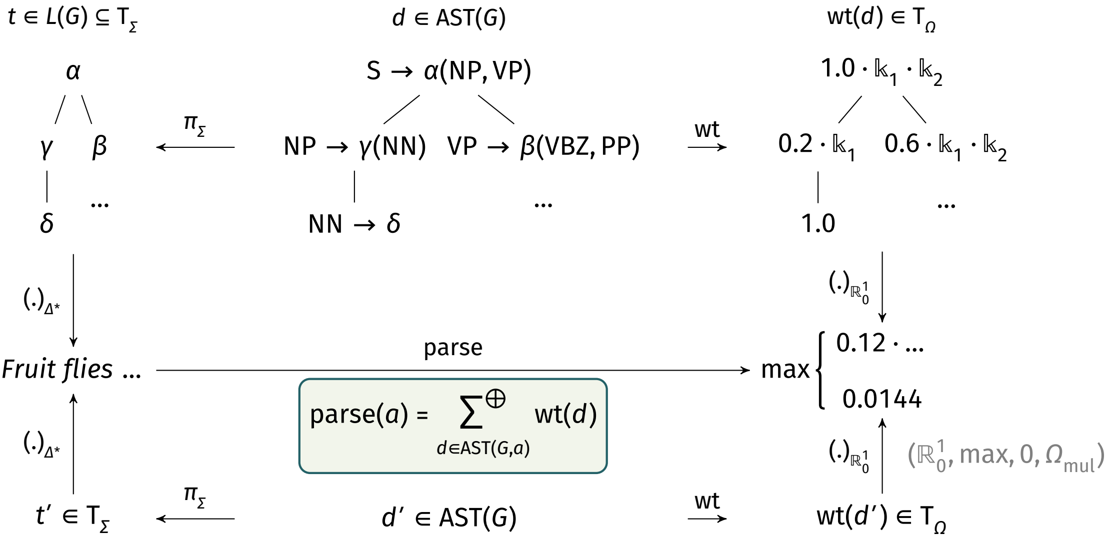

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The M-monoid parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)

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- wRTG-LM $\big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$
- $a \in \mathcal{L}$

$$\mathrm{parse}(a) = \sum^{\oplus}_{d \in \mathrm{AST}(G,a)} \mathrm{wt}(d)$$

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Two-phase pipeline (Goodman 1999; Nederhof 2003)

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→ canonical weighted deduction system

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$\longrightarrow$ canonical weighted deduction system $\longrightarrow$ wRTG-LM $((G', \mathcal{CFG}^{\emptyset}), \mathbb{K}, \mathrm{wt}')$

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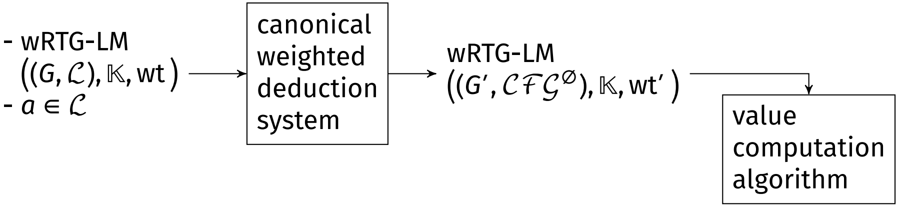

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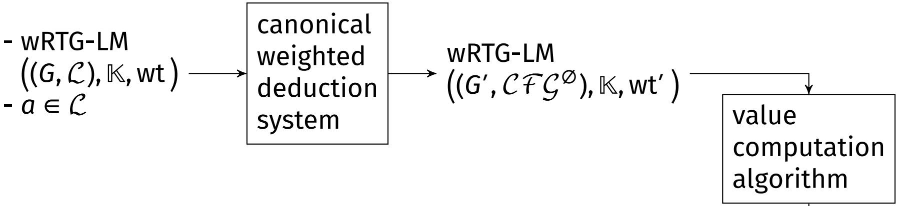

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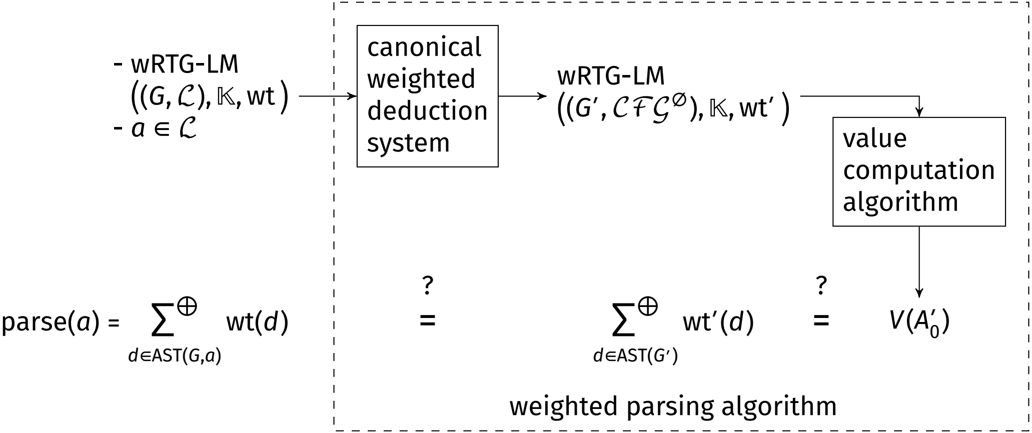

The canonical weighted deduction system

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$\xrightarrow{\text{cwds}}$ wRTG-LM $\big((G', \mathcal{CFG}^{\emptyset}), \mathbb{K}, \mathrm{wt}'\big)$

Parsing as deduction (Shieber, Schabes, and Pereira 1995)

$$\frac{[A_1, a_1] \;\dots\; [A_m, a_m]}{[A, a_0]} \quad \begin{cases} A \to \sigma(A_1, \dots, A_m) \text{ is a rule in } G \\ a_0, a_1, \dots, a_m \text{ are part of a decomposition of } a \text{ in } \mathcal{L} \\ a_0 = \phi(\sigma)(a_1, \dots, a_m) \end{cases}$$

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- wRTG-LM $((G, \mathcal{L}), \mathbb{K}, \mathrm{wt})$
- $a \in \mathcal{L}$

$$\xrightarrow{\text{cwds}} \text{wRTG-LM } ((G', \mathcal{CFG}^{\emptyset}), \mathbb{K}, \mathrm{wt}')$$

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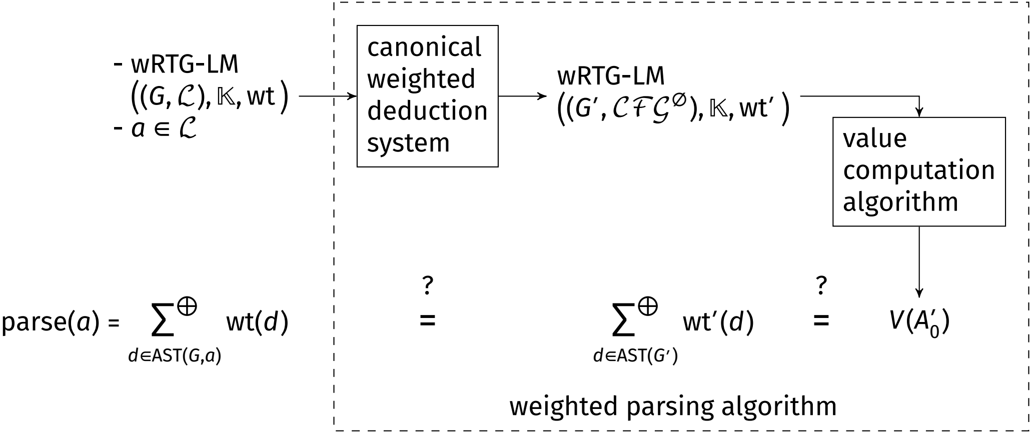
The M-monoid parsing algorithm

Two-phase pipeline (Goodman 1999; Nederhof 2003)



The value computation algorithm

- Infinite summation vs. termination

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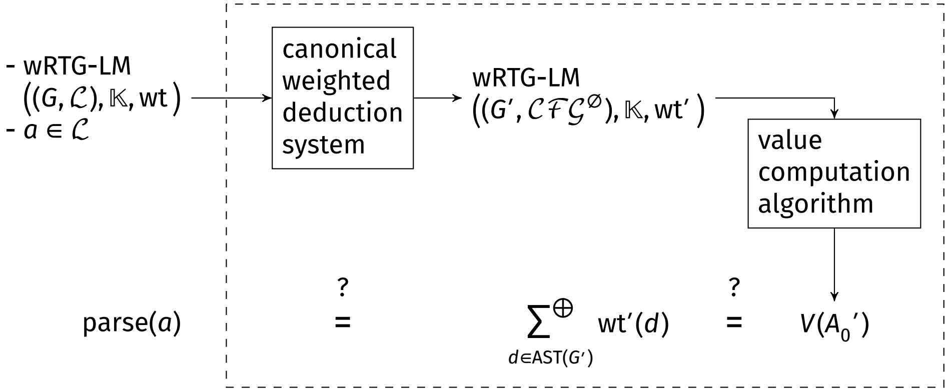
- Generalization to our framework
  - introduce branching (strings ⇝ trees)
  - requires distributivity

Application scenarios



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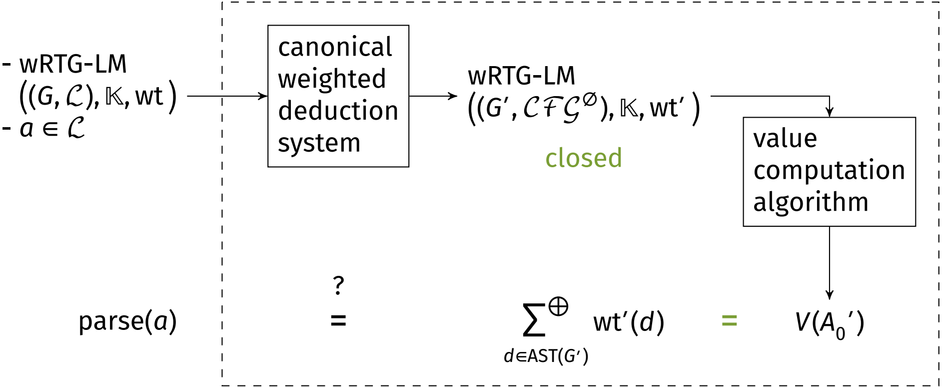

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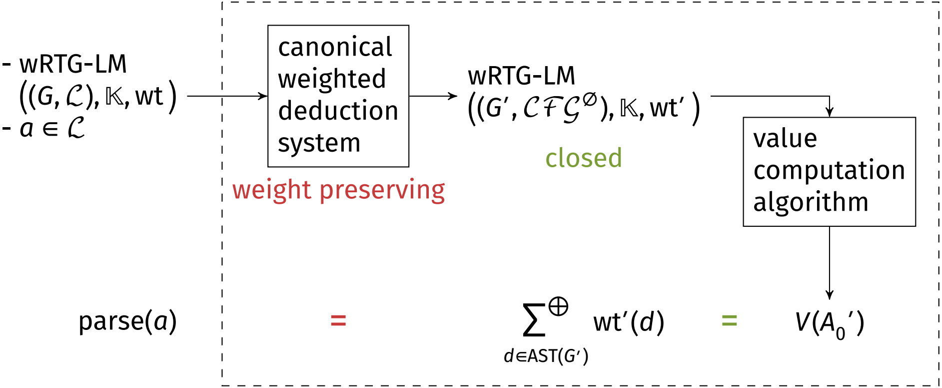

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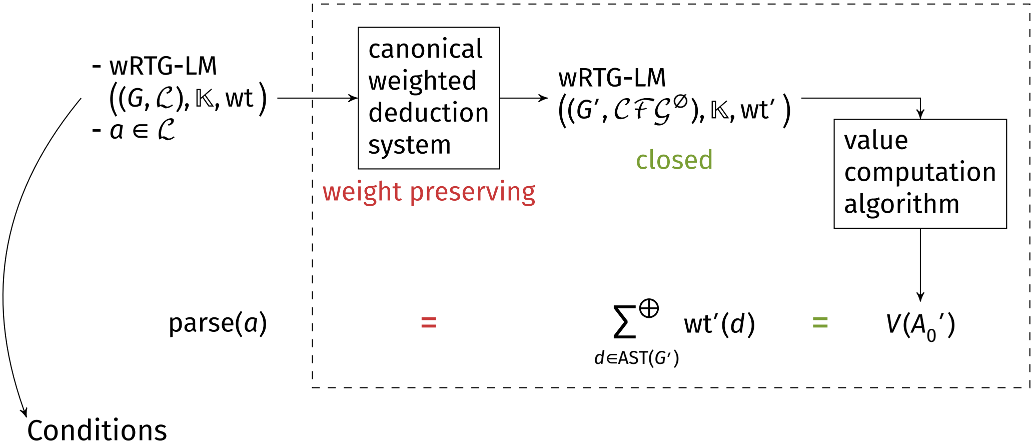

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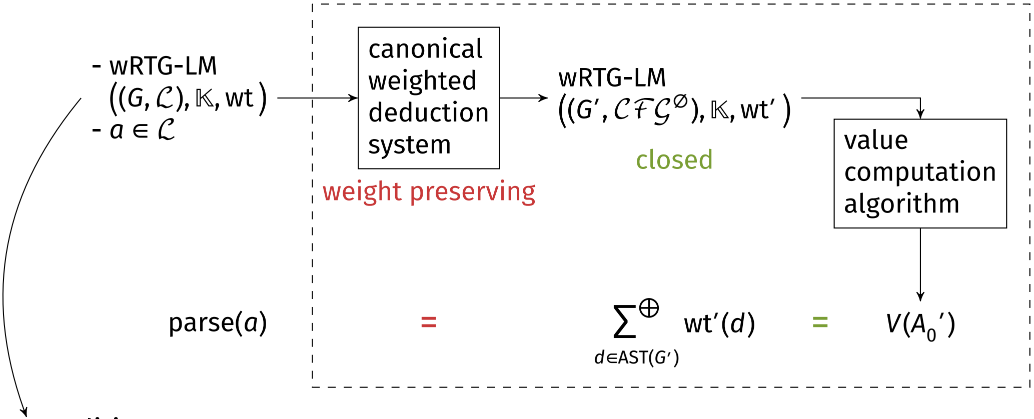

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- Sufficient: $((G, \mathcal{L}), \mathbb{K}, \mathrm{wt})$ is closed or nonlooping

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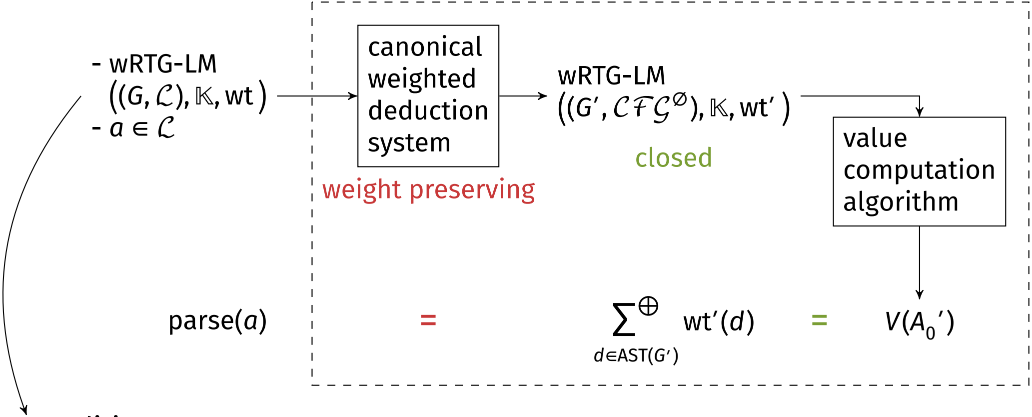

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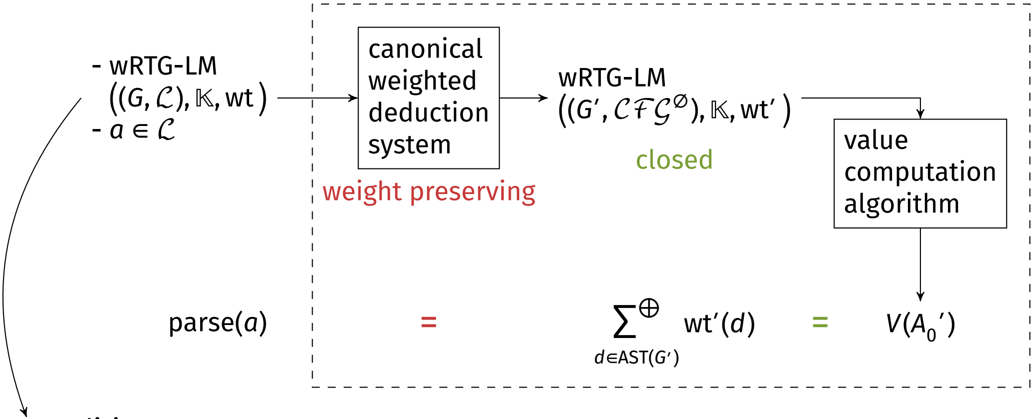

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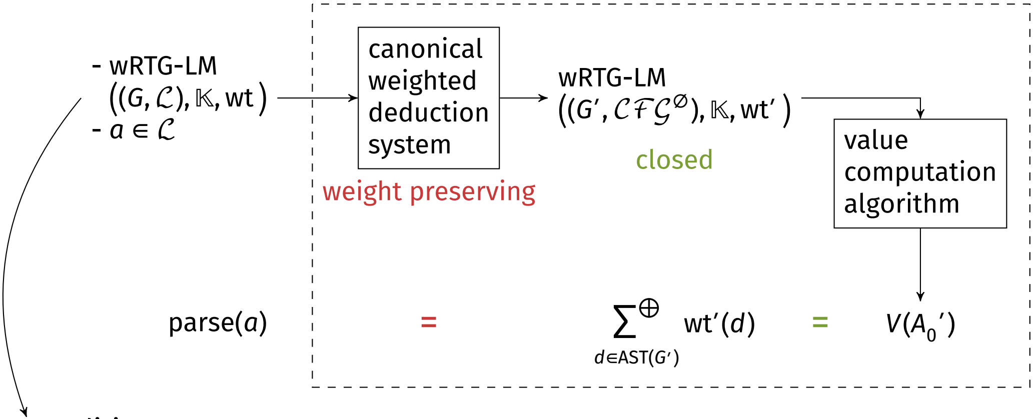

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  e.g., CFG, LCFRS, TAG

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Canonical weighted deduction system

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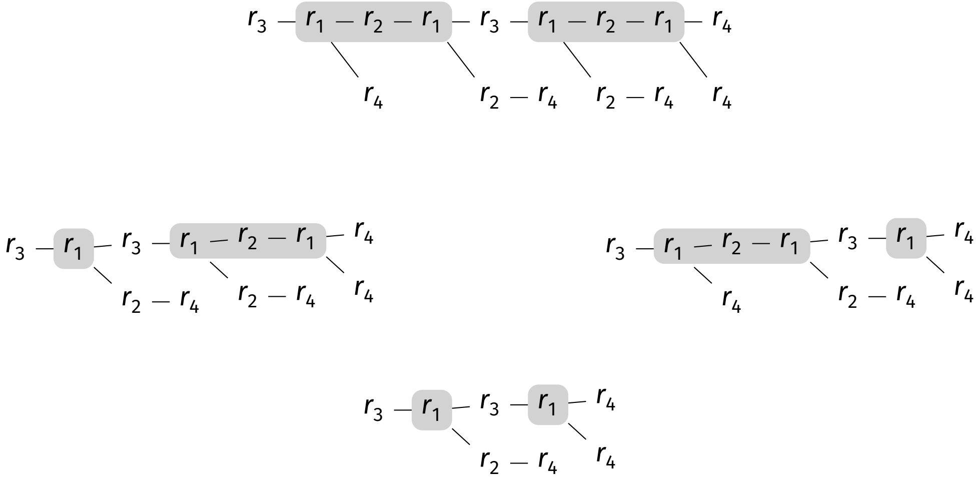
Closed wRTG-LMs

cutout(d, ρ)



Closed wRTG-LMs

Definition

Let $c \in \mathbb{N}$. A wRTG-LM $\mathcal{G} = \big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$ is *c-closed* if $\mathbb{K}$ is distributive and d-complete, and for each $d \in \mathrm{T}_R$ and cyclic string $\rho \in R^*$ the following holds: if there is a (c, ρ)-cyclic path in d, then

$$\mathrm{wt}(d)_{\mathbb{K}} \oplus \bigoplus_{d \in \mathrm{cutout}(d,\rho)} \mathrm{wt}(d)_{\mathbb{K}} = \bigoplus_{d \in \mathrm{cutout}(d,\rho)} \mathrm{wt}(d)_{\mathbb{K}} \ .$$

$\mathrm{AST}(G)^{(c)}$: each cycle at most c times

↓ closed, distributive, d-complete

Theorem

For every $c \in \mathbb{N}$ and c-closed wRTG-LM $\big((G, \mathcal{L}), \mathbb{K}, \mathrm{wt}\big)$ the following holds:

$$\sum^{\oplus}_{d \in \mathrm{AST}(G')} \mathrm{wt}(d)_{\mathbb{K}} = \bigoplus_{d \in \mathrm{AST}(G)^{(c)}} \mathrm{wt}(d)_{\mathbb{K}} \ .$$

The value computation algorithm

Input: a wRTG-LM $\big((G', \mathcal{CFG}^{\varnothing}), (\mathbb{K}, \oplus, \mathbb{0}, \Omega), \mathrm{wt}'\big)$ with $G' = (N', \Sigma', A'_0, R')$
Variables: $V\colon N' \to \mathbb{K}$, $V_{\text{new}} \in \mathbb{K}$, $\mathit{changed} \in \mathbb{B}$
Output: $V(A'_0)$

for each $A \in N'$ **do**
  $V(A) \leftarrow \mathbb{0}$
repeat
  $\mathit{changed} \leftarrow$ false
  for each $A \in N'$ **do**
    $V_{\text{new}} \leftarrow \mathbb{0}$
    for each $r = (A \to \langle x_1 \dots x_m \rangle(A_1, \dots, A_m))$ in R' **do**
      $V_{\text{new}} \leftarrow V_{\text{new}} \oplus \mathrm{wt}'(r)\big(V(A_1), \dots, V(A_m)\big)$
    if $V(A) \neq V_{\text{new}}$ **then**
      $\mathit{changed} \leftarrow$ true
    $V(A) \leftarrow V_{\text{new}}$
until $\mathit{changed}$ = false