
Max-plus automata
and
Cost-register automata

WATA 2021

Laure Daviaud
CitAI, City University of London

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- More open questions than known results
- Specific to max-plus but some results are general
- Work in progress with Andrew Ryzhikov (City University) and Filip Mazowiecki (MPI)
- Let's try to make it a bit interactive

A poll to start with

Go to: **Pollev.com/lauredaviaud279**

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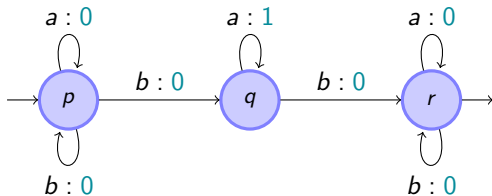
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You are:

A listening

B not listening

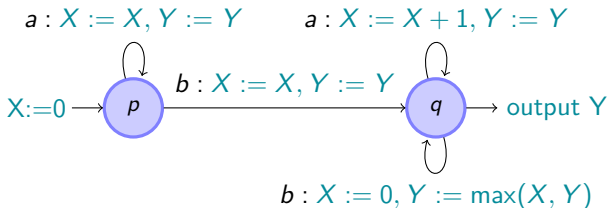
Max-Plus Automata



Go to: [PollEv.com/lauredaviaud279](https://www.poll-ev.com/lauredaviaud279)

What is the value of the word $a^3 b^2 a^3 b^4 a^5 bab^3 a^6$?

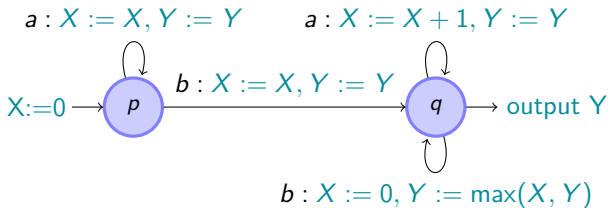
Linear Cost Register Automata



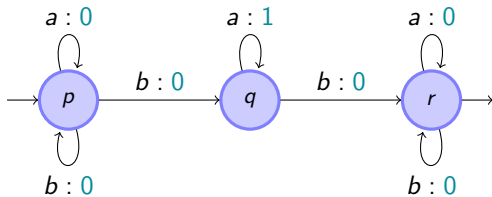
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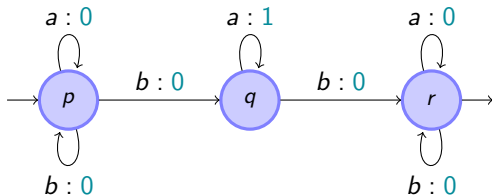
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Linear Cost Register Automata for Max-plus

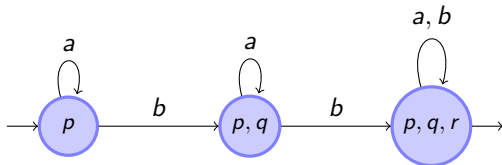


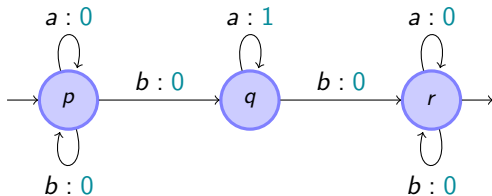
- Deterministic
- Registers X, Y, Z, \dots starting with value 0
- Updates of the form: $X := \max(X + c, Y + d, Z + e, \dots)$
- Output of the form: $\max(X + c, Y + d, Z + e, \dots)$



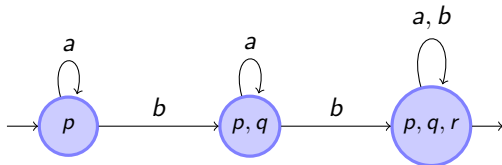


- Subset construction to get a deterministic automaton



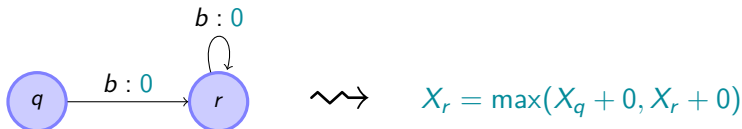


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- Registers X_p , X_q , X_r to track the values in the corresponding states

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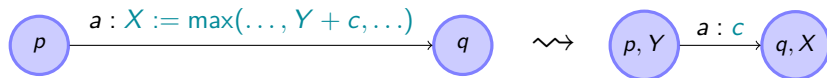
A linear CRA with states p, q, r, \dots and registers X, Y, Z, \dots

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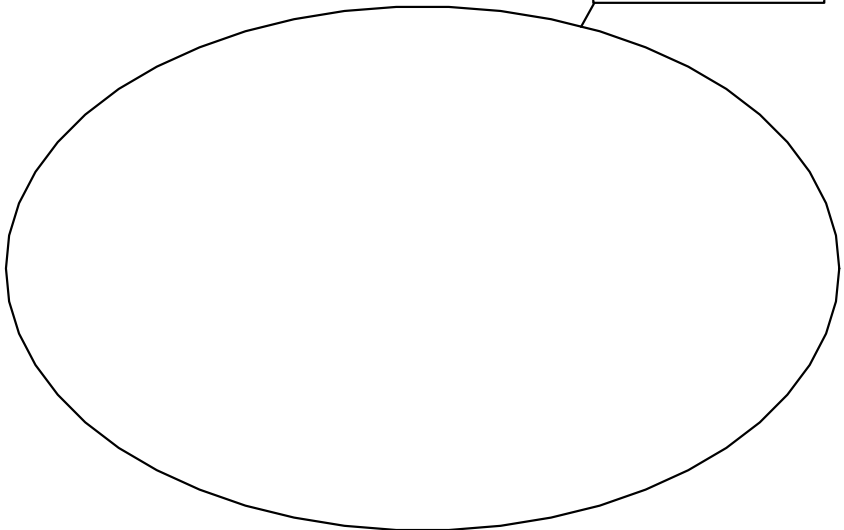
\rightsquigarrow A Max-plus automaton with states $(p, X), (p, Y), (p, Z), (q, X), \dots$

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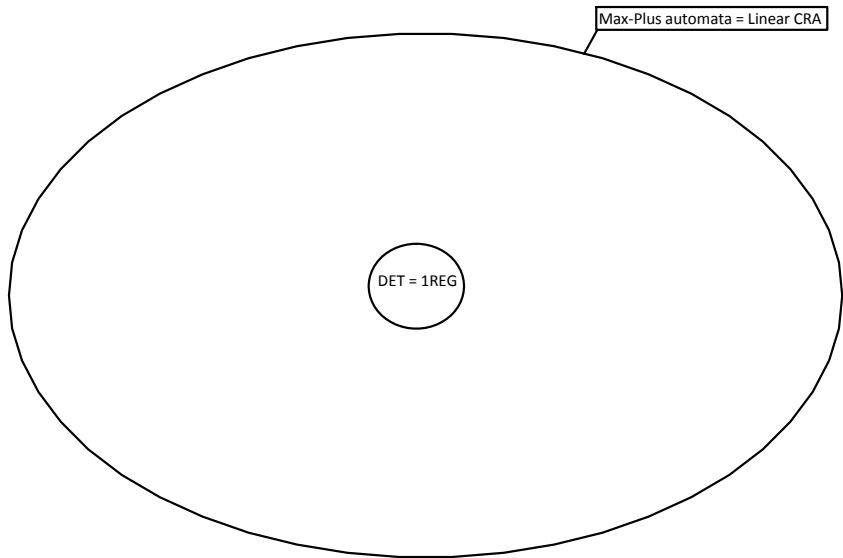


Hierarchy - a start



Max-Plus automata = Linear CRA

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Max-Plus automata = Linear CRA

Undecidable!

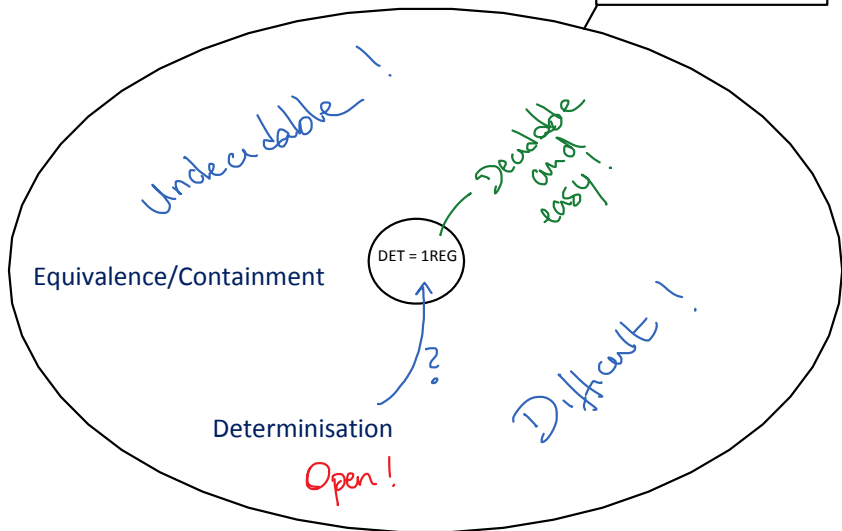
DET = 1REG

Decidable
and
easy!

Difficult!

Hierarchy - a start

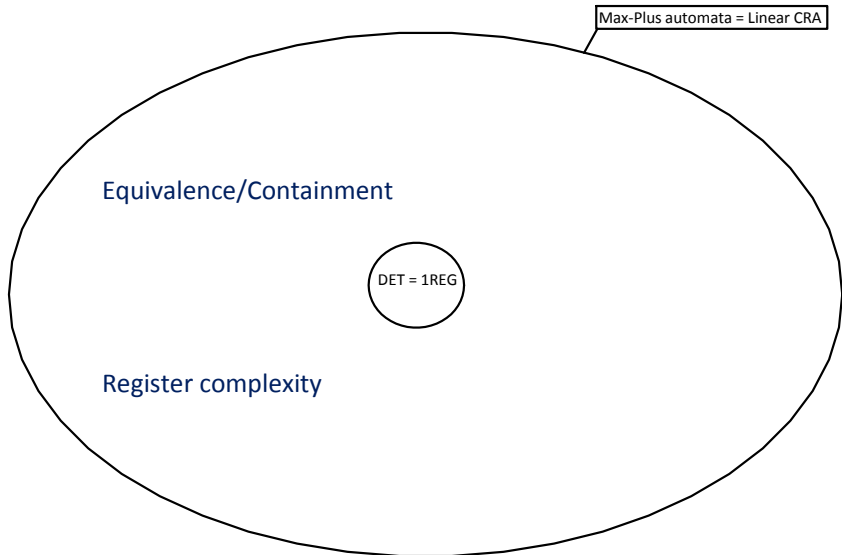
Max-Plus automata = Linear CRA



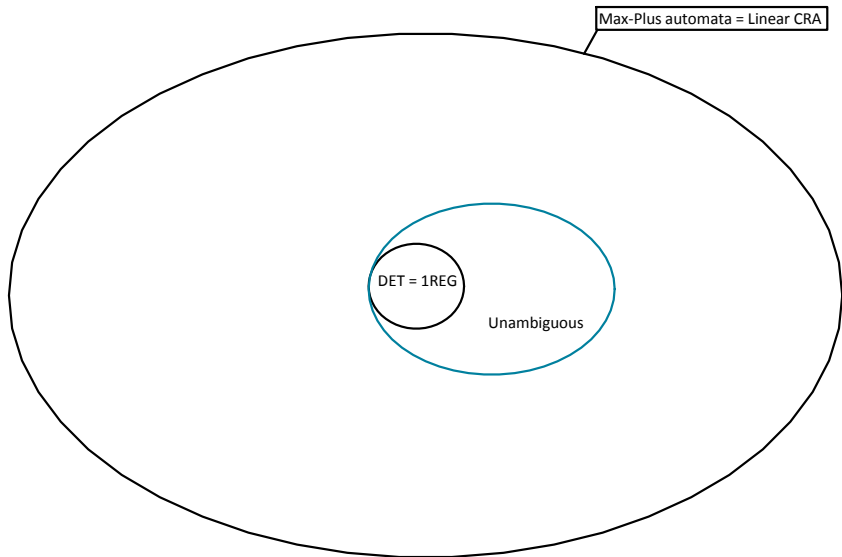
Register Complexity

Given a max-plus automaton \mathcal{A} and an integer k ,
is \mathcal{A} equivalent to a linear cost register automaton with k
registers?

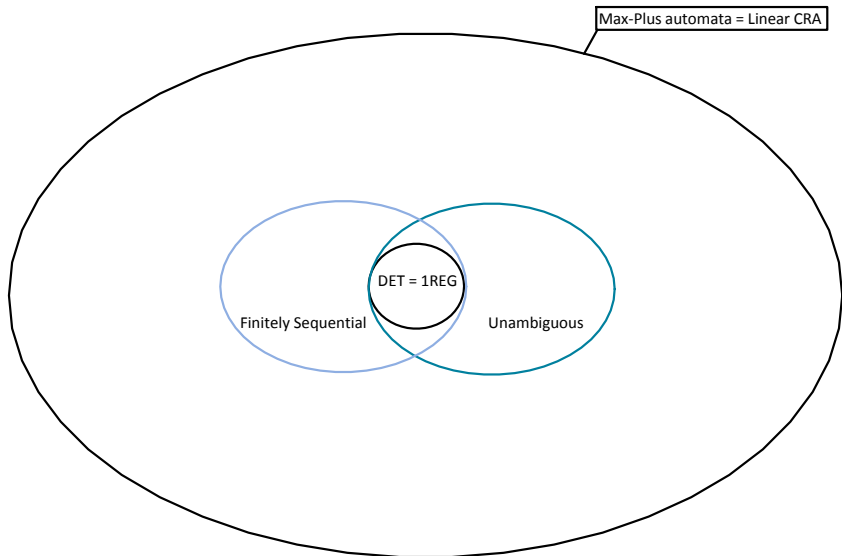
Hierarchy - what happens in the middle?



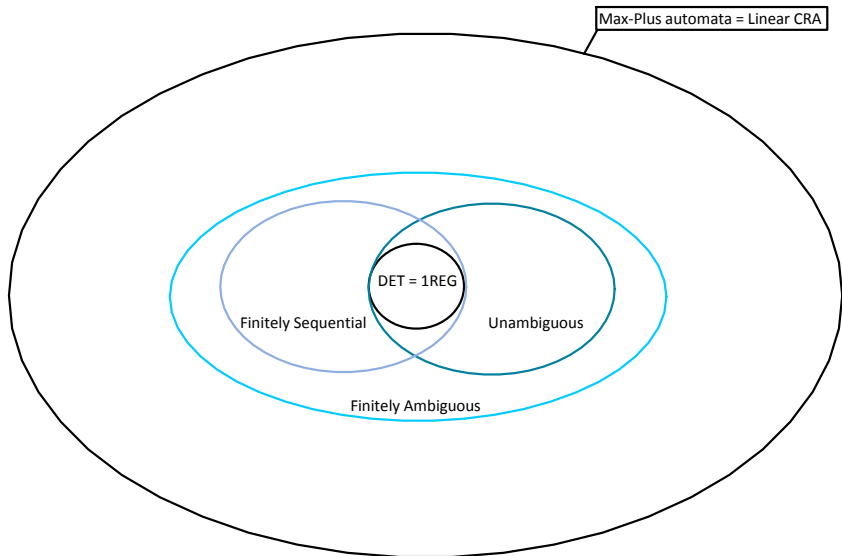
Ambiguity



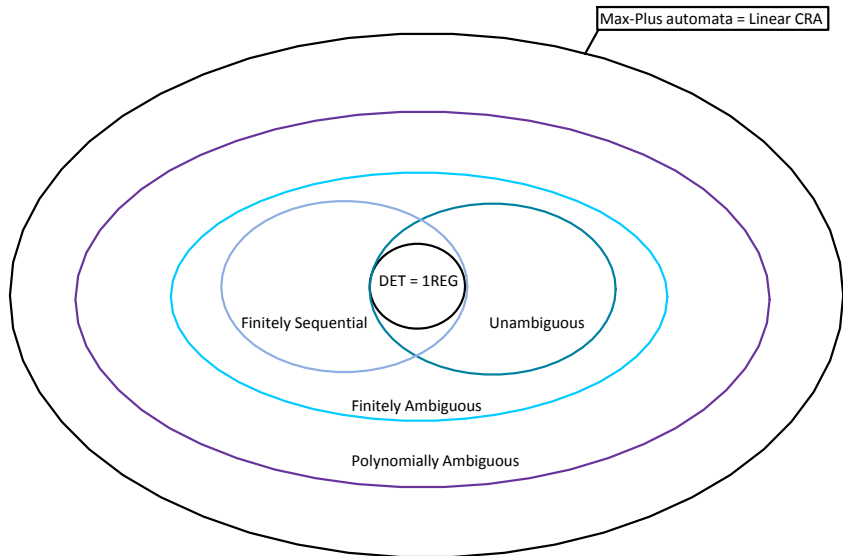
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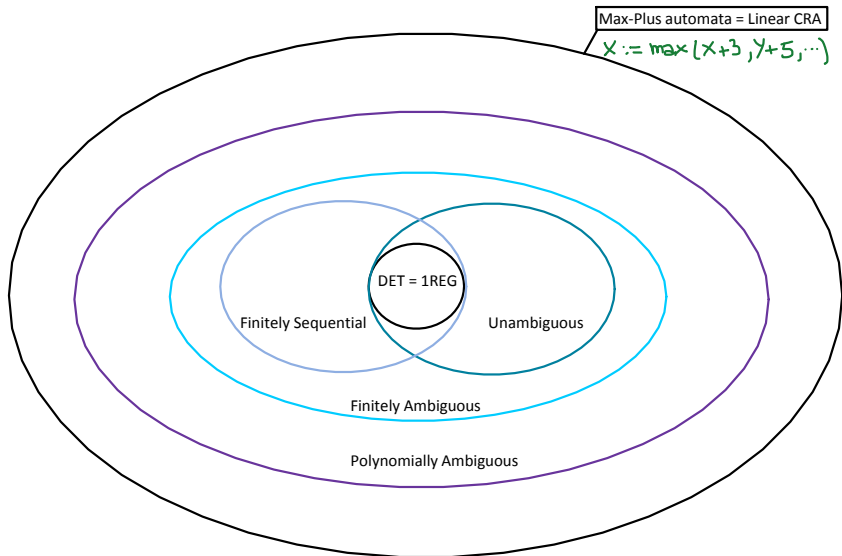
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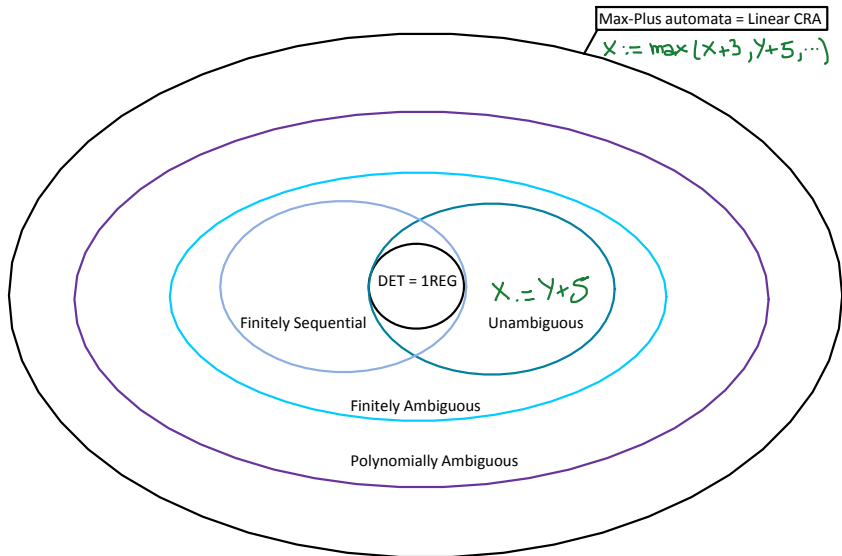
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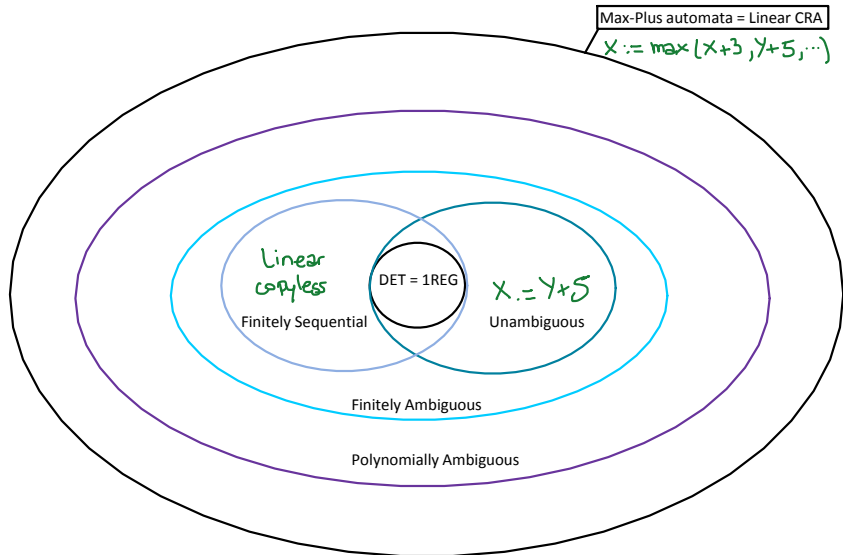
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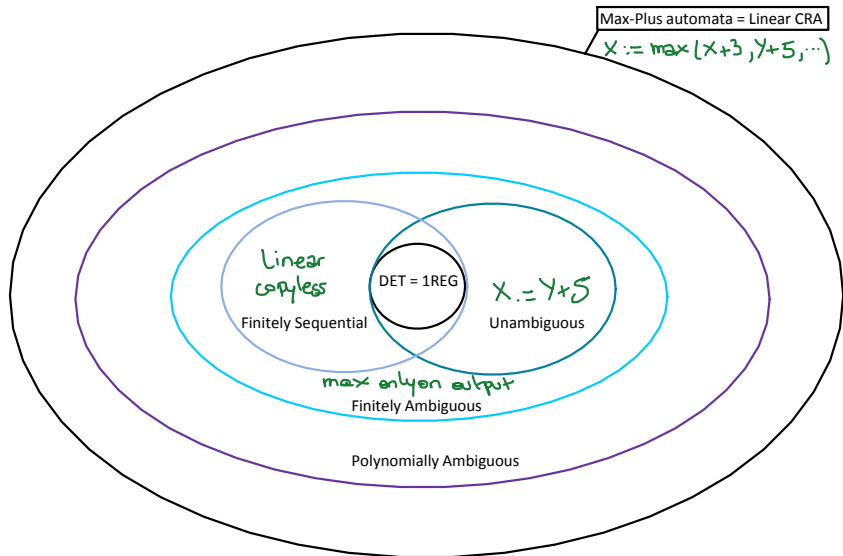
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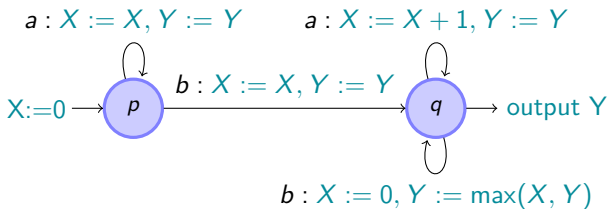
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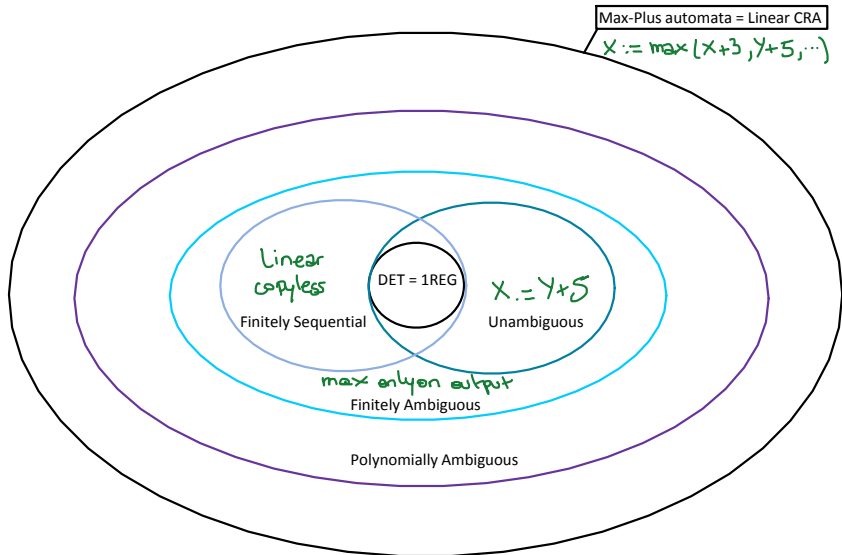
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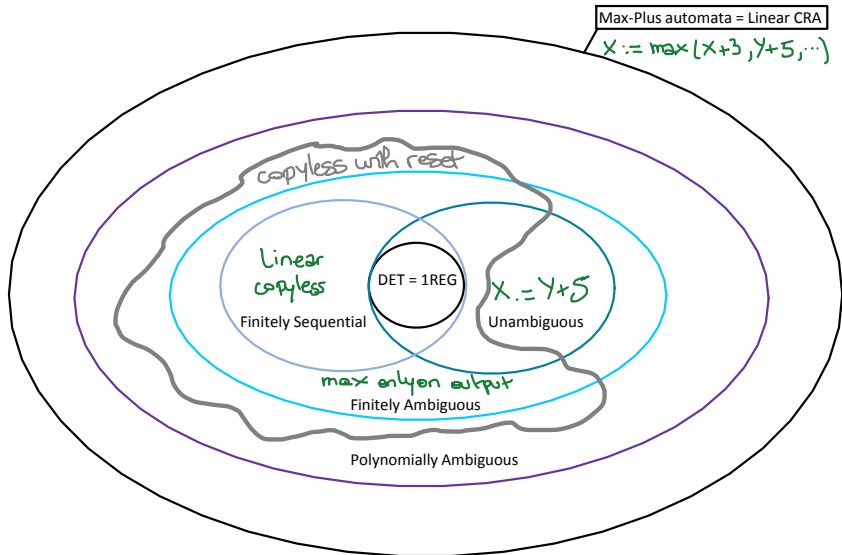
Reset



Other classes



Other classes



Some examples

Alphabet $\{a, b, c\}$

$$w_1 c w_2 c \cdots c w_k \mapsto \ell_1 + \cdots + \ell_k$$

where ℓ_i is the number of a in w_i if w_i ends with an a , and the number of b in w_i otherwise.

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This function belongs to:

- A Unambiguous
- B Finitely ambiguous (not unambiguous)
- C Copyless Linear CRA with reset

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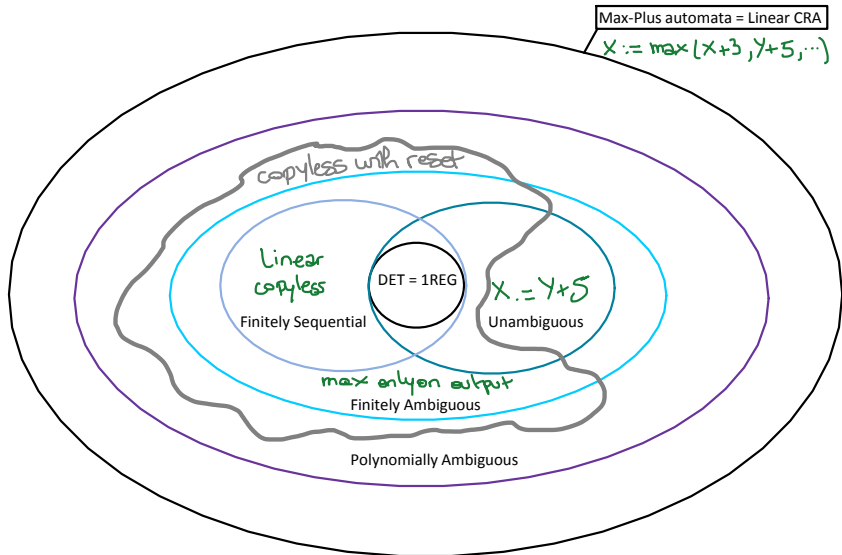
$$a^{\ell_1} b a^{\ell_2} b \cdots b a^{\ell_k} \mapsto \max(\ell_1, \dots, \ell_k)$$

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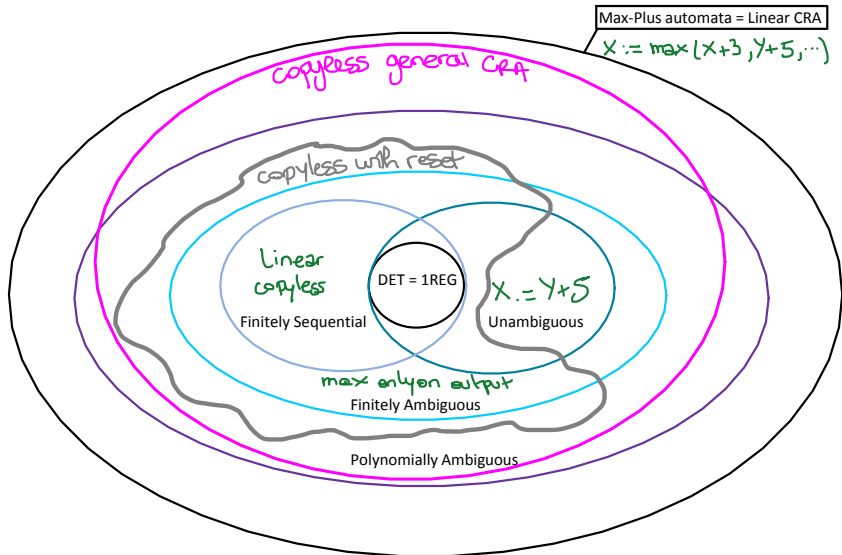
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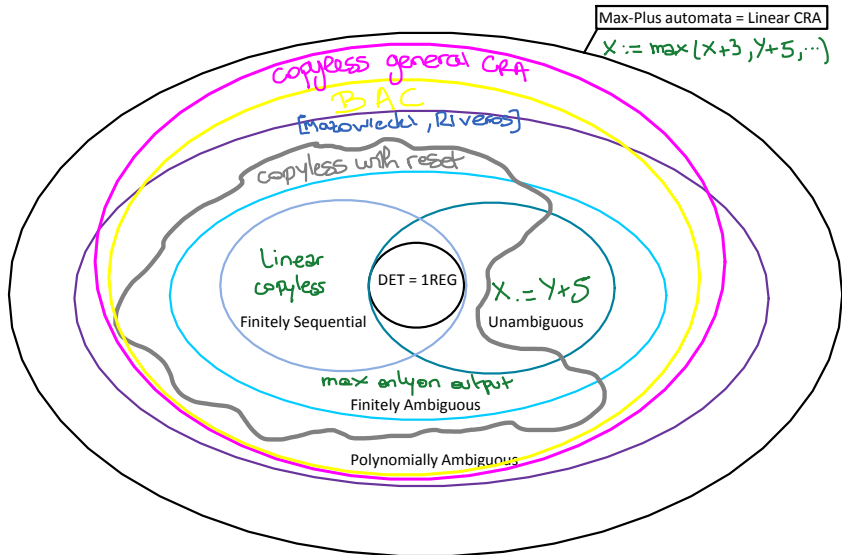
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- Characterisations with Max-plus Automata/Cost Register Automata

Hierarchy

