Finite Sequentiality of Max-Plus Tree Automata

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Max-Plus Automata

$$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_3 \xrightarrow{a} q_4 \xrightarrow{b} q_5 \xrightarrow{a} q_6$$

Weights in $\mathbb{R} \cup \{-\infty\}$

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Weight of run:

initial weight + transition weights + final weight

Weight of word:

maximum over all runs

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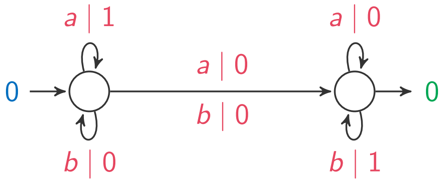

Max-Plus Automata: Ambiguity

one "initial state"

sequential / deterministic — no two valid $p \xrightarrow{a} q_1,\ p \xrightarrow{a} q_2$

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unambiguous — $|\mathrm{Run}(w)| \leq 1$

finitely ambiguous — $|\mathrm{Run}(w)| \leq M$

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decidable if

- $\mathcal{A}$ unambiguous (Bala, Koniński)
- $\mathcal{A}$ finitely ambiguous (Bala)

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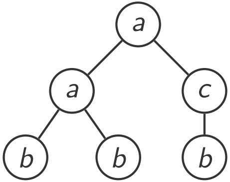

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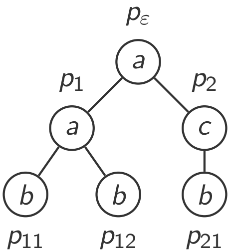

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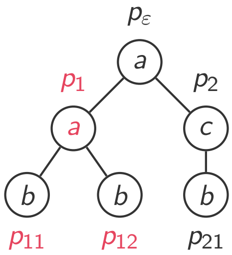
(p_{11}, p_{12}, a, p_1)



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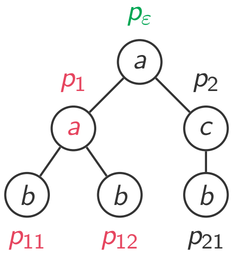

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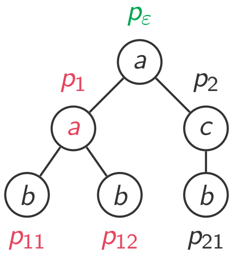

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THM Finite Sequentiality decidable for finitely ambiguous max-plus tree automata

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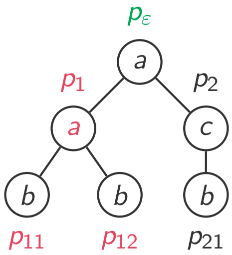

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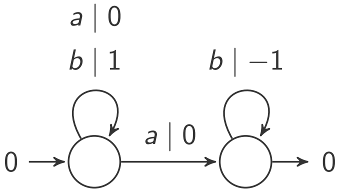

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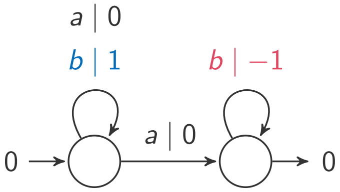

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$\Updownarrow$

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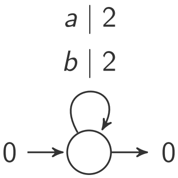

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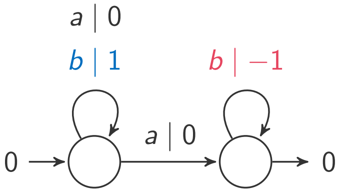

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- Büchi's Theorem (MSO logic $\leftrightarrow$ tree automata)