Weighted linear dynamic logic with two-sorted semantics

Gustav Grabolle

WATA2021
April 22, 2021

The essential logic: LDL

Linear Dynamic Logic on finite traces [De Giacomo, Vardi, 2013]

- intuitive syntax (= PDL)
- characterizes regular languages (> LTL)
- satisfiability in PSPACE (= LTL)

The essential logic: LDL

Linear Dynamic Logic on finite traces [De Giacomo, Vardi, 2013]

- intuitive syntax (= PDL)
- characterizes regular languages (> LTL)
- satisfiability in PSPACE (= LTL)

The essential logic: LDL

Linear **D**ynamic **L**ogic on finite traces [De Giacomo, Vardi, 2013]

- intuitive syntax (= PDL)
- characterizes regular languages (> LTL)
- satisfiability in PSPACE (= LTL)

The essential logic: LDL

Linear **D**ynamic **L**ogic on finite traces [De Giacomo, Vardi, 2013]

- intuitive syntax (= PDL)
- characterizes regular languages (> LTL)
- satisfiability in PSPACE (= LTL)

How to LDL

Syntax

$$\begin{array}{lcll} \varphi & = & p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho> \varphi & \text{state formula} \\ \rho & = & a \mid \varphi? \mid \rho + \rho \mid \rho \,;\, \rho \mid \rho^* & \text{transition formula} \end{array}$$

Semantics

How to LDL

Syntax

$$\begin{array}{lcll} \varphi & = & p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi & \text{state formula} \\ \rho & = & a \mid \varphi? \mid \rho + \rho \mid \rho\,;\rho \mid \rho^* & \text{transition formula} \end{array}$$

Semantics

How to LDL

Syntax

$$\begin{array}{lcll} \varphi & = & p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi & \text{state formula} \\ \rho & = & a \mid \varphi? \mid \rho + \rho \mid \rho\,;\rho \mid \rho^* & \text{transition formula} \end{array}$$

Semantics

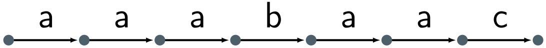
How to LDL

Syntax

$$\varphi \quad = \quad p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho> \varphi$$ state formula

$$\rho \quad = \quad a \mid \varphi? \mid \rho + \rho \mid \rho\,;\rho \mid \rho^*$$ transition formula

Semantics

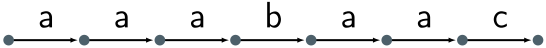


How to LDL

Syntax

$$\varphi \;=\; p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi \quad \text{state formula}$$
$$\rho \;=\; a \mid \varphi? \mid \rho + \rho \mid \rho\,;\rho \mid \rho^* \quad \text{transition formula}$$

Semantics

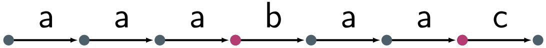


$p_b \vee p_c$

Syntax

$$\varphi \quad = \quad p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi \qquad \text{state formula}$$
$$\rho \quad = \quad a \mid \varphi? \mid \rho + \rho \mid \rho \,;\, \rho \mid \rho^* \qquad \text{transition formula}$$

Semantics



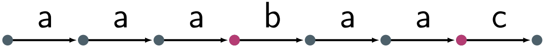
$p_b \vee p_c$

How to LDL

Syntax

$$\begin{aligned} \varphi &= p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi && \text{state formula} \\ \rho &= a \mid \varphi? \mid \rho + \rho \mid \rho\,;\rho \mid \rho^* && \text{transition formula} \end{aligned}$$

Semantics

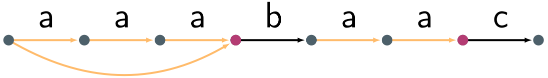


$a\,;(a\,;a)^*$ $\quad p_b \vee p_c$

Syntax

$$\varphi \;=\; p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi \qquad \text{state formula}$$
$$\rho \;=\; a \mid \varphi? \mid \rho + \rho \mid \rho\,;\rho \mid \rho^* \qquad \text{transition formula}$$

Semantics



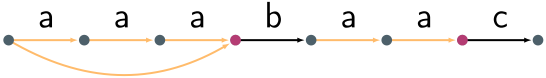
$a\,;(a\,;a)^*$ $\quad p_b \vee p_c$

How to LDL

Syntax

$$\varphi \;=\; p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi \qquad \text{state formula}$$
$$\rho \;=\; a \mid \varphi? \mid \rho + \rho \mid \rho\,;\rho \mid \rho^* \qquad \text{transition formula}$$

Semantics



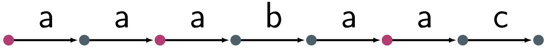
$$<a\,;(a\,;a)^*>\, p_b \vee p_c$$

How to LDL

Syntax

$$\begin{aligned} \varphi &= p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi && \text{state formula} \\ \rho &= a \mid \varphi? \mid \rho + \rho \mid \rho \,;\, \rho \mid \rho^* && \text{transition formula} \end{aligned}$$

Semantics



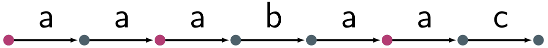
$$<a\,;(a\,;a)^*>\, p_b \vee p_c$$

How to LDL

Syntax

$$
\begin{array}{lcll}
\varphi & = & p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho> \varphi & \text{state formula} \\
\rho & = & a \mid \varphi? \mid \rho + \rho \mid \rho \,;\, \rho \mid \rho^* & \text{transition formula}
\end{array}
$$

Semantics



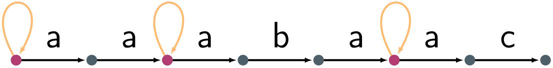
$$\left(<a\,;\,(a\,;\,a)^*> p_b \vee p_c\right)?$$

How to LDL

Syntax

$$\begin{aligned} \varphi &= p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho> \varphi \quad \text{state formula} \\ \rho &= a \mid \varphi? \mid \rho + \rho \mid \rho \,;\, \rho \mid \rho^* \quad \text{transition formula} \end{aligned}$$

Semantics

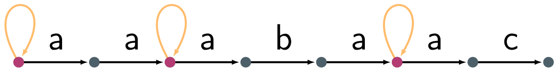


$$\left(<a\,;\,(a\,;\,a)^*> p_b \vee p_c\right)?$$

Syntax

$$\varphi \quad = \quad p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi \qquad \text{state formula}$$
$$\rho \quad = \quad a \mid \varphi? \mid \rho + \rho \mid \rho\,;\rho \mid \rho^* \qquad \text{transition formula}$$

Semantics



$$\left(<a\,;(a\,;a)^*> p_b \vee p_c\right)?$$

ρ rational

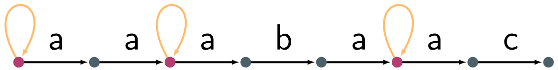
How to LDL

Syntax

$$\varphi \quad = \quad p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho> \varphi$$ state formula

$$\rho \quad = \quad a \mid \varphi? \mid \rho + \rho \mid \rho \,;\, \rho \mid \rho^*$$ transition formula

Semantics



$$\left(< a \,;\, (a \,;\, a)^* > p_b \vee p_c \right)?$$

ρ rational $\quad + \quad$ $L(\neg\varphi) = L(\varphi)^C$

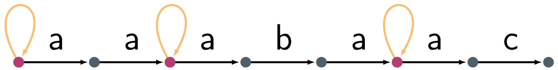
How to LDL

Syntax

$$\varphi \;=\; p_a \mid \neg\varphi \mid \varphi \vee \varphi \mid <\rho>\varphi \qquad \text{state formula}$$

$$\rho \;=\; a \mid \varphi? \mid \rho + \rho \mid \rho \,;\, \rho \mid \rho^* \qquad \text{transition formula}$$

Semantics



$$\left(<a\,;\,(a\,;\,a)^*>p_b \vee p_c\right)?$$

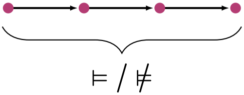
ρ rational $\quad + \quad L(\neg\varphi) = L(\varphi)^C \quad + \quad \varphi U \psi \equiv <(\varphi?;a)^*>\psi$

Lets talk about weights

non-weighted

weighted

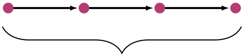
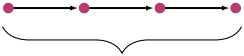
word



relation

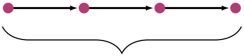
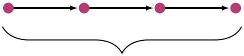
Two-sorted weighted semantics

Lets talk about weights

	non-weighted	weighted
word	 $\models$ / $\not\models$	 $s(w) \in K$
relation		

Two-sorted weighted semantics

Lets talk about weights

	non-weighted	weighted
word	 $\models / \not\models$	 $s(w) \in K$
relation		

Two-sorted weighted semantics

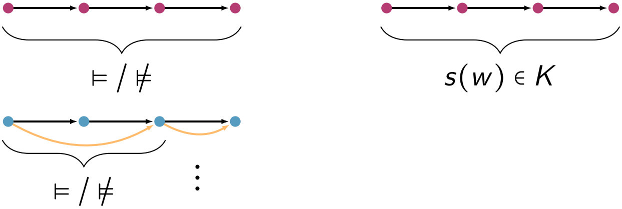
$$\|\varphi\| : A^* \to K$$

Lets talk about weights

non-weighted weighted

word

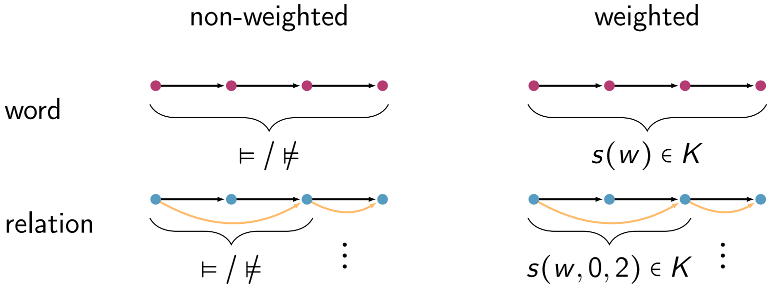
relation



Two-sorted weighted semantics

$$\|\varphi\| : A^* \to K$$

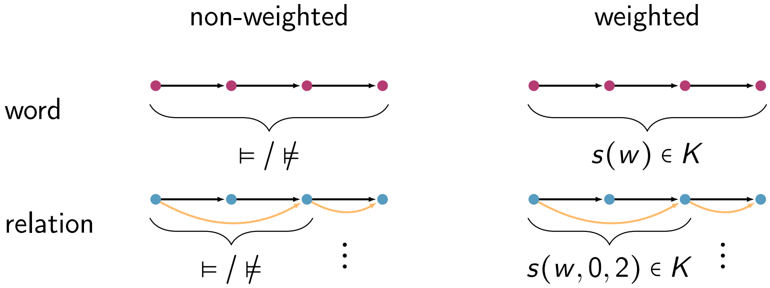
Lets talk about weights



Two-sorted weighted semantics

$$\|\varphi\| : A^* \to K$$

Lets talk about weights



Two-sorted weighted semantics

$$\|\varphi\| : A^* \to K \qquad (\!(\rho)\!) : A^* \times (\mathbb{N} \times \mathbb{N}) \to K$$

Two-sorted weighted linear dynamic logic

Syntax of 2LDL(A,K)

$$\begin{array}{lcll} \varphi & = & \phi \mid k \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid <\rho> \varphi & \text{state formula} \\ \rho & = & a \mid \varphi? \mid \rho \oplus \rho \mid \rho \,;\, \rho \mid \rho^{\oplus} & \text{transition formula} \end{array}$$

Two-sorted weighted linear dynamic logic

Syntax of 2LDL(A,K)

$$
\begin{array}{lcll}
\varphi & = & \phi \mid k \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid <\rho> \varphi & \text{state formula} \\
\rho & = & a \mid \varphi ? \mid \rho \oplus \rho \mid \rho \,;\, \rho \mid \rho^{\oplus} & \text{transition formula}
\end{array}
$$

$$
R_{\rho;\lambda}(w) \quad = \quad \{(l, r) \mid \quad \exists m.(l, m) \in R_\rho(w) \wedge (m, r) \in R_\lambda(w)\}
$$

Two-sorted weighted linear dynamic logic

Syntax of 2LDL(A,K)

$$\varphi = \phi \mid k \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid <\rho> \varphi \quad \text{state formula}$$
$$\rho = a \mid \varphi? \mid \rho \oplus \rho \mid \rho \,;\, \rho \mid \rho^{\oplus} \quad \text{transition formula}$$

$$R_{\rho;\lambda}(w) = \{(l,r) \mid \exists m.(l,m) \in R_\rho(w) \wedge (m,r) \in R_\lambda(w)\}$$

$$(\!(\rho\,;\,\lambda)\!)(w,l,r) = \sum_{l \leq m \leq r} (\!(\rho)\!)(w,l,m) \cdot (\!(\lambda)\!)(w,m,r)$$

Two-sorted weighted linear dynamic logic

Syntax of 2LDL(A,K)

$$\varphi \quad = \quad \phi \mid k \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid <\rho> \varphi \qquad \text{state formula}$$

$$\rho \quad = \quad a \mid \varphi? \mid \rho \oplus \rho \mid \rho \,;\, \rho \mid \rho^{\oplus} \qquad \text{transition formula}$$

$$R_{\rho;\lambda}(w) \quad = \quad \{(l, r) \mid \quad \exists m.(l, m) \in R_\rho(w) \wedge (m, r) \in R_\lambda(w)\}$$

$$(\!(\rho \,;\, \lambda)\!)(w, l, r) \quad = \quad \sum_{l \leq m \leq r} (\!(\rho)\!)(w, l, m) \cdot (\!(\lambda)\!)(w, m, r)$$

Two-sorted weighted linear dynamic logic

Syntax of 2LDL(A,K)

$$\varphi \;=\; \phi \mid k \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid <\rho> \varphi \qquad \text{state formula}$$

$$\rho \;=\; a \mid \varphi? \mid \rho \oplus \rho \mid \rho \,;\, \rho \mid \rho^{\oplus} \qquad \text{transition formula}$$

$$R_{\rho;\lambda}(w) \;=\; \{(l, r) \mid \exists m.(l, m) \in R_\rho(w) \wedge (m, r) \in R_\lambda(w)\}$$

$$(\!(\rho\,;\,\lambda)\!)(w, l, r) \;=\; \sum_{l \leq m \leq r} (\!(\rho)\!)(w, l, m) \cdot (\!(\lambda)\!)(w, m, r)$$

Two-sorted weighted linear dynamic logic

Syntax of 2LDL(A,K)

$$\begin{array}{lcll} \varphi & = & \phi \mid k \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid <\rho> \varphi & \text{state formula} \\ \rho & = & a \mid \varphi? \mid \rho \oplus \rho \mid \rho \,;\, \rho \mid \rho^{\oplus} & \text{transition formula} \end{array}$$

$$R_{\rho;\lambda}(w) = \{(l, r) \mid \exists m.(l, m) \in R_\rho(w) \wedge (m, r) \in R_\lambda(w)\}$$

$$(\!(\rho \,;\, \lambda)\!)(w, l, r) = \sum_{l \leq m \leq r} (\!(\rho)\!)(w, l, m) \cdot (\!(\lambda)\!)(w, m, r)$$

Two-sorted weighted linear dynamic logic

Syntax of 2LDL(A,K)

$$\begin{array}{lcll} \varphi & = & \phi \mid k \mid \varphi \oplus \varphi \mid \varphi \otimes \varphi \mid <\rho> \varphi & \text{state formula} \\ \rho & = & a \mid \varphi? \mid \rho \oplus \rho \mid \rho \,;\, \rho \mid \rho^{\oplus} & \text{transition formula} \end{array}$$

$$R_{\rho;\lambda}(w) = \{(l, r) \mid \exists m.(l, m) \in R_\rho(w) \wedge (m, r) \in R_\lambda(w)\}$$

$$(\!(\rho\,;\,\lambda)\!)(w, l, r) = \sum_{l \leq m \leq r} (\!(\rho)\!)(w, l, m) \cdot (\!(\lambda)\!)(w, m, r)$$

$$(\!(\varphi?)\!)(w, l, r) = \begin{cases} \|\varphi\|(w_{\geq l}) & \text{if } l = r \\ 0 & \text{otherwise} \end{cases}$$

The power of questions

The power of questions

$$\varphi_1 = <(2?\,;a)^{\oplus}> \phi_{\varepsilon} \qquad \|\varphi_1\|(a^n) = 2^n$$

The power of questions

$$\varphi_1 = <(2? \,;\, a)^{\oplus}> \phi_\varepsilon \qquad \|\varphi_1\|(a^n) = 2^n$$

$$\varphi_2 = <(\varphi_1? \,;\, a)^{\oplus}> \phi_\varepsilon \qquad \|\varphi_2\|(a^n) = 2^n \cdot 2^{n-1} \cdot \ldots = 2^{\frac{n(n+1)}{2}}$$

The power of questions

$$\varphi_1 = <(2? \,;\, a)^{\oplus}> \phi_\varepsilon \qquad \|\varphi_1\|(a^n) = 2^n$$

$$\varphi_2 = <(\varphi_1? \,;\, a)^{\oplus}> \phi_\varepsilon \qquad \|\varphi_2\|(a^n) = 2^n \cdot 2^{n-1} \cdot \ldots = 2^{\frac{n(n+1)}{2}}$$

$$\vdots \qquad\qquad \vdots$$

The power of questions

$$\varphi_1 = \; <(2? \,;\, a)^{\oplus}> \phi_{\varepsilon} \qquad \|\varphi_1\|(a^n) = 2^n$$

$$\varphi_2 = \; <(\varphi_1? \,;\, a)^{\oplus}> \phi_{\varepsilon} \qquad \|\varphi_2\|(a^n) = 2^n \cdot 2^{n-1} \cdot \ldots = 2^{\frac{n(n+1)}{2}}$$

$$\vdots \qquad\qquad\qquad \vdots$$

Testing hierarchy for LDL

$$2\mathrm{LDL}_0 \subseteq wRAT = 2LDL_1 \subseteq 2LDL_2 \subseteq \ldots$$

The power of questions

$$\varphi_1 = <(2? \,;\, a)^{\oplus}> \phi_\varepsilon \qquad \|\varphi_1\|(a^n) = 2^n$$

$$\varphi_2 = <(\varphi_1? \,;\, a)^{\oplus}> \phi_\varepsilon \qquad \|\varphi_2\|(a^n) = 2^n \cdot 2^{n-1} \cdot \ldots = 2^{\frac{n(n+1)}{2}}$$

$$\vdots \qquad\qquad \vdots$$

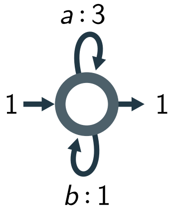
Testing hierarchy for LDL

$$2\mathrm{LDL}_0 \subsetneq wRAT = 2LDL_1 \subsetneq 2LDL_2 \subsetneq \ldots$$

strict for $\mathbb{N}, \mathbb{Q}, \mathbb{R}, \ldots$

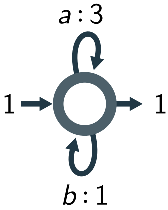
WLNA: Automata with automata with ...

$\mathcal{B}$:

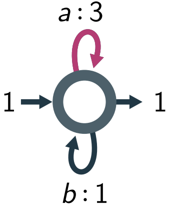


WLNA: Automata with automata with ...

$\mathcal{B}$:

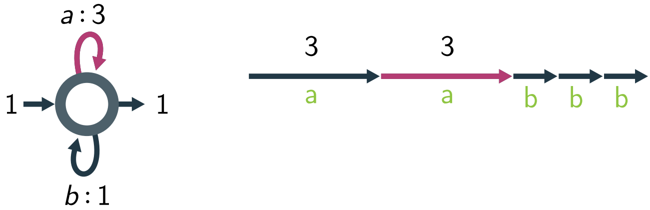


$\mathcal{B}$:



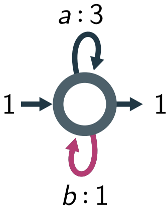
WLNA: Automata with automata with ...

$\mathcal{B}$:



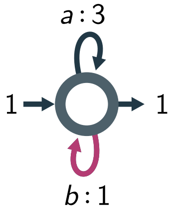
WLNA: Automata with automata with ...

$\mathcal{B}$:



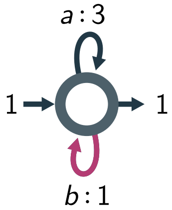
WLNA: Automata with automata with ...

$\mathcal{B}$:



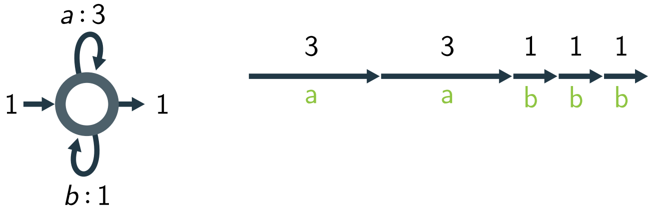
WLNA: Automata with automata with ...

$\mathcal{B}$:



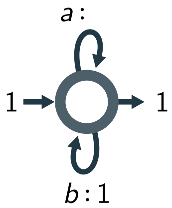
WLNA: Automata with automata with ...

$\mathcal{B}$:

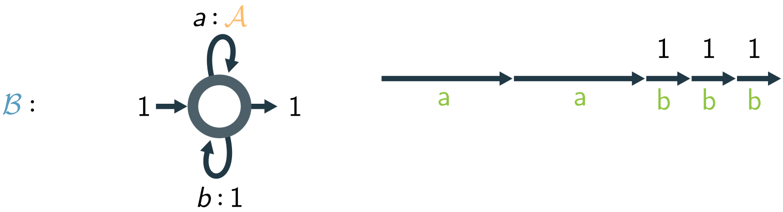


WLNA: Automata with automata with ...

$\mathcal{B}$:



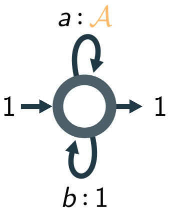
WLNA: Automata with automata with ...



WLNA: Automata with automata with ...

$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

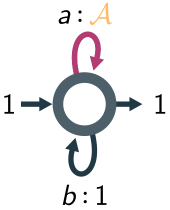
$\mathcal{B}$:



WLNA: Automata with automata with ...

$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

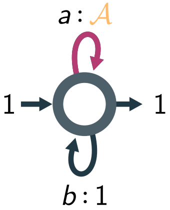
$\mathcal{B}$:



WLNA: Automata with automata with ...

$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

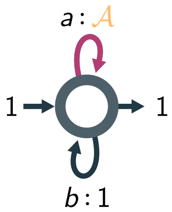
$\mathcal{B}$:



WLNA: Automata with automata with ...

$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

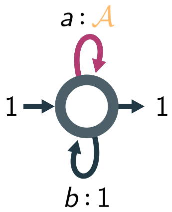
$\mathcal{B}$:

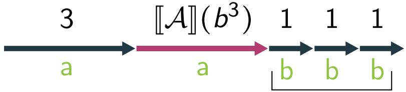


WLNA: Automata with automata with ...

$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

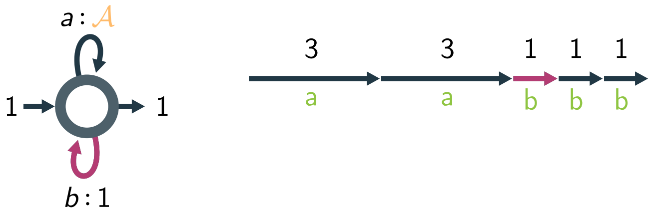
$\mathcal{B}$:





$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

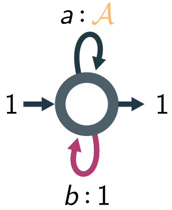
$\mathcal{B}$:



WLNA: Automata with automata with ...

$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

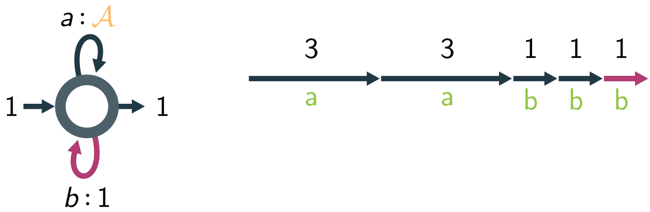
$\mathcal{B}$:



WLNA: Automata with automata with ...

$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

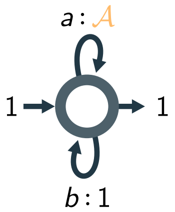
$\mathcal{B}$:



WLNA: Automata with automata with ...

$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

$\mathcal{B}$:

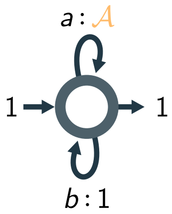


$[\![\mathcal{B}]\!](a^m b^n) = n^m$

WLNA: Automata with automata with ...

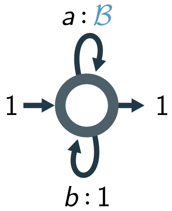
$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

$\mathcal{B}$:



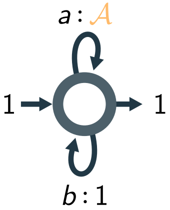
$[\![\mathcal{B}]\!](a^m b^n) = n^m$

$\mathcal{C}$:



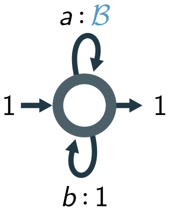
$\mathcal{A}$: weighted automaton $[\![\mathcal{A}]\!](a^m b^n) = n$

$\mathcal{B}$:



$[\![\mathcal{B}]\!](a^m b^n) = n^m$

$\mathcal{C}$:



$[\![\mathcal{C}]\!](a^m b^n) = n^{\frac{(m-1)\cdot m}{2}}$

The automata formerly known as ...

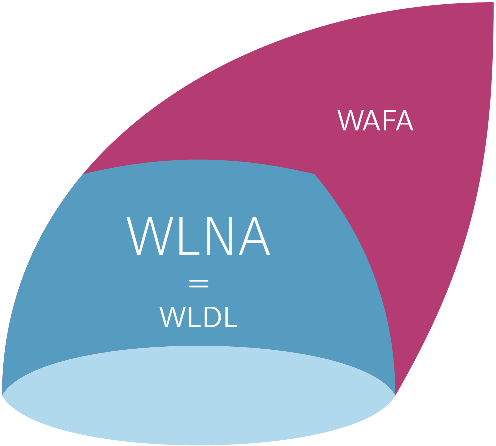
WLNA

=

WLDL

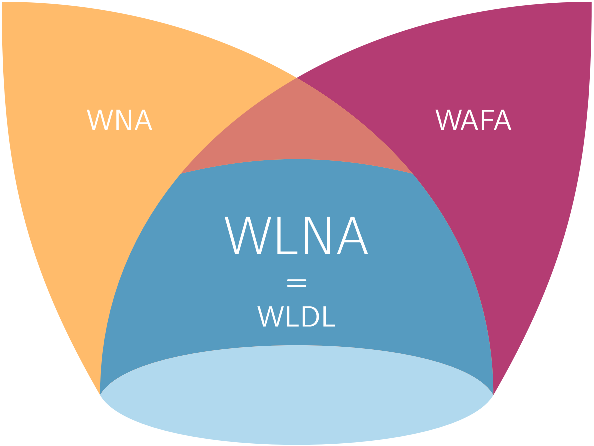
The automata formerly known as …

The automata formerly known as ...



P. Kostolányi, F. Mišún: Alternating weighted automata over commutative semirings, Theoretical Computer Science 740 (2018)

The automata formerly known as ...



P. Kostolányi, F. Mišún: Alternating weighted automata over commutative semirings, Theoretical Computer Science 740 (2018)

B. Bollig, P. Gastin, B. Monmege, M. Zeitoun: Pebble weighted automata and transitive closure logics, ICALP (2010)

The devil is in the distinctions

WLDL

2LDL

$$<\rho\,;\lambda>\varphi$$
$$=$$
$$<\rho><\lambda>\varphi$$

WFA

WLTL

EQUALITY in $\bar{\mathbb{Q}}$

D. Manfred, G. Rahonis: Weighted linear dynamic logic (2016)

The devil is in the distinctions

	WLDL	2LDL
$<\rho\,;\lambda>\varphi$ $=$ $<\rho><\lambda>\varphi$	No	Yes
WFA		
WLTL		
EQUALITY in $\bar{\mathbb{Q}}$		

D. Manfred, G. Rahonis: Weighted linear dynamic logic (2016)

The devil is in the distinctions

	WLDL	2LDL
$<\rho\,;\lambda>\varphi$ $=$ $<\rho><\lambda>\varphi$	No	Yes
WFA	$=$	$\geq$
WLTL		
EQUALITY in $\bar{\mathbb{Q}}$		

D. Manfred, G. Rahonis: Weighted linear dynamic logic (2016)

The devil is in the distinctions

	WLDL	2LDL
$<\rho\,;\lambda>\varphi$ $=$ $<\rho><\lambda>\varphi$	No	Yes
WFA	$=$	$\geq$
WLTL	$\ngeq, \nleq$	$\geq$
EQUALITY in $\bar{\mathbb{Q}}$		

D. Manfred, G. Rahonis: Weighted linear dynamic logic (2016)

The devil is in the distinctions

	WLDL	2LDL
$<\rho\,;\lambda>\varphi$ $=$ $<\rho><\lambda>\varphi$	No	Yes
WFA	$=$	$\geq$
WLTL	$\ngeq, \nleq$	$\geq$
EQUALITY in $\bar{\mathbb{Q}}$	2EXPTIME	$\leq$ ACKERMANN

D. Manfred, G. Rahonis: Weighted linear dynamic logic (2016)

Decisions

ZERONESS is decidable over $\bar{\mathbb{Q}}$

Input: $\varphi \in \mathrm{2LDL}(A, \bar{\mathbb{Q}})$
Output: Yes/No if $\|\varphi\| = 0 / \|\varphi\| \neq 0$

...is decidable in ACKERMANN

ZERONESS is decidable over $\bar{\mathbb{Q}}$

Input: $\varphi \in 2\mathsf{LDL}(A, \bar{\mathbb{Q}})$
Output: Yes/No if $\|\varphi\| = 0 / \|\varphi\| \neq 0$

...is decidable in ACKERMANN

$$\varphi \in 2\mathsf{LDL}(A, \bar{\mathbb{Q}}) \quad \Rightarrow \quad \exists \mathcal{A}_L \in \mathsf{WLNA}(A, \bar{\mathbb{Q}}).(\|\mathcal{A}\| = \|\varphi\|)$$

ZERONESS is decidable over $\bar{\mathbb{Q}}$

Input: $\varphi \in 2\mathrm{LDL}(A, \bar{\mathbb{Q}})$
Output: Yes/No if $\|\varphi\| = 0 / \|\varphi\| \neq 0$

...is decidable in ACKERMANN

$$\begin{aligned}\varphi \in 2\mathrm{LDL}(A, \bar{\mathbb{Q}}) &\Rightarrow \exists \mathcal{A}_L \in \mathrm{WLNA}(A, \bar{\mathbb{Q}}).(\|\mathcal{A}\| = \|\varphi\|) \\ &\Rightarrow \exists \mathcal{A}_A \in \mathrm{WAFA}(A, \bar{\mathbb{Q}}).(\|\mathcal{A}_L\| = \|\mathcal{A}_A\|)\end{aligned}$$

Decisions

ZERONESS is decidable over $\bar{\mathbb{Q}}$

Input: $\varphi \in 2\mathsf{LDL}(A, \bar{\mathbb{Q}})$
Output: Yes/No if $\|\varphi\| = 0 / \|\varphi\| \neq 0$

...is decidable in ACKERMANN

$$\begin{aligned}
\varphi \in 2\mathsf{LDL}(A, \bar{\mathbb{Q}}) &\Rightarrow \exists \mathcal{A}_L \in \mathsf{WLNA}(A, \bar{\mathbb{Q}}).(\|\mathcal{A}\| = \|\varphi\|) \\
&\Rightarrow \exists \mathcal{A}_A \in \mathsf{WAFA}(A, \bar{\mathbb{Q}}).(\|\mathcal{A}_L\| = \|\mathcal{A}_A\|) \\
&\Rightarrow \exists \mathcal{A}_P \in \mathsf{polyA}(A, \bar{\mathbb{Q}}).(\|\mathcal{A}_P\| = \|\mathcal{A}_A\|^R)
\end{aligned}$$

Decisions

ZERONESS is decidable over $\bar{\mathbb{Q}}$

Input: $\varphi \in 2\mathrm{LDL}(A, \bar{\mathbb{Q}})$
Output: Yes/No if $\|\varphi\| = 0 / \|\varphi\| \neq 0$

...is decidable in ACKERMANN

$$\begin{aligned} \varphi \in 2\mathrm{LDL}(A, \bar{\mathbb{Q}}) &\Rightarrow \exists \mathcal{A}_L \in \mathrm{WLNA}(A, \bar{\mathbb{Q}}).(\|\mathcal{A}\| = \|\varphi\|) \\ &\Rightarrow \exists \mathcal{A}_A \in \mathrm{WAFA}(A, \bar{\mathbb{Q}}).(\|\mathcal{A}_L\| = \|\mathcal{A}_A\|) \\ &\Rightarrow \exists \mathcal{A}_P \in \mathrm{polyA}(A, \bar{\mathbb{Q}}).(\|\mathcal{A}_P\| = \|\mathcal{A}_A\|^R) \end{aligned}$$

ZERONESS for $\mathrm{polyA}(A, \bar{\mathbb{Q}})$ is ACKERMANN-hard

M. Benedikt, et al.: Polynomial automata: Zeroness and applications, 32nd Annual ACM/IEEE Symposium on Logic in Computer Science (LICS), 2017