

Decidability of crisp-determinization for weighted finite automata over past-finite monotonic strong bimonoids

M. Droste^a Z. Fülöp^{b1} D. Kószó^{b2} H. Vogler^c

^a*Institute of Computer Science, University of Leipzig*

^b*Department of Foundations of Computer Science, University of Szeged*

^c*Faculty of Computer Science, Technische Universität Dresden*

Abstract: A general problem in the theory of automata is the question whether an automaton is determinizable, i.e., whether we can construct an equivalent deterministic automaton. While each classical finite automaton is determinizable, this is not true for weighted finite automata (wfa). In fact, there is a nondeterministic wfa over the field \mathbb{Q} which is not determinizable (cf. [2]). Thus, it is natural to identify classes of wfa which have the following property: for each wfa \mathcal{A} of that class it is decidable whether \mathcal{A} is determinizable. If a class has this property (respectively, does not have this property), then we say that the determinization problem for this class is decidable (respectively, undecidable).

We consider the crisp-determinization problem for the class of wfa over strong bimonoids [3,5]. A wfa is crisp-deterministic if it is deterministic, and each of its transitions carries either the additive or multiplicative unit of the strong bimonoid; weights different from these units may only appear as initial and final weights. In [6] it was shown that the crisp-determinization problem for the class of wfa over strong bimonoids is undecidable.

In this presentation, we deal with wfa over past-finite monotonic strong bimonoids. Such algebras are based on monotonic semirings [1] and they share many properties with the semiring of natural numbers (strong kind of well-foundedness, monotonicity of operations, zero-sum and zero-divisor free, partial order on the carrier set), however distributivity is not required and also does not follow from the axioms. We obtain a general characterization of when trim wfa over past-finite monotonic strong bimonoids are crisp-determinizable. As a consequence, we show that the crisp-determinization problem for the class of (a) wfa over additively locally finite and past-finite monotonic strong bimonoids and (b) unambiguous wfa over past-finite monotonic strong bimonoids is decidable [4]. In this case, assuming the strong bimonoid to be computable, the equivalent crisp-deterministic wfa can be computed. Moreover, we show that for each wfa \mathcal{A} over a past-finite monotonic strong bimonoid and a natural number k it is decidable whether the cardinality of the image of the semantics of \mathcal{A} is bounded by k .

References

- [1] B. Borchardt, Z. Fülöp, Z. Gazdag, and A. Maletti. Bounds for tree automata with polynomial costs. *J. Autom. Lang. Comb.*, 10:107–157, 2005.
- [2] B. Borchardt and H. Vogler. Determinization of finite state weighted tree automata. *J. Autom. Lang. Comb.*, 8(3):417–463, 2003.
- [3] M. Čirić, M. Droste, J. Ignjatović, and H. Vogler. Determinization of weighted finite automata over strong bimonoids. *Inf. Sci.*, 180(18):3479–3520, 2010.
- [4] M. Droste, Z. Fülöp, D. Kószó, and H. Vogler. Crisp-Determinization of Weighted Tree Automata over Additively Locally Finite and Past-Finite Monotonic Strong Bimonoids Is Decidable. G. Jirásková and G. Pighizzini (Eds.): DCFS 2020, LNCS 12442, pp. 39-51, 2020.
- [5] M. Droste, T. Stüber, and H. Vogler. Weighted finite automata over strong bimonoids. *Inf. Sci.*, 180(1):156–166, 2010.
- [6] Z. Fülöp, D. Kószó, and H. Vogler. Crisp-determinization of weighted tree automata over strong bimonoids. see also: arXiv:1912.02660v2 [cs.FL] 2 Feb 2021.

¹Research of this author was supported by grant TUDFO/47138-1/2019-ITM of the Ministry for Innovation and Technology, Hungary.

²Supported by the ÚNKP-19-3 and ÚNKP-20-3 - New National Excellence Program of the Ministry for Innovation and Technology from the source of the National Research, Development and Innovation Fund.