Posets with Interfaces
(Extended Abstract)

Uli Fahrenberg[1], Christian Johansen[2], Georg Struth[3], and Ratan Bahadur Thapa[2]

[1] École Polytechnique, Palaiseau, France
[2] University of Oslo, Norway
[3] University of Sheffield, UK

This work is inspired by Tony Hoare's programme of building graph models of concurrent Kleene algebra (CKA) [5] for real-world applications. CKA extends the sequential compositions, nondeterministic choices and unbounded finite iterations of imperative programs modelled by Kleene algebra into concurrency, adding operations of parallel composition and iteration, and a weak interchange law for the sequential-parallel interaction. Such algebras have a long history in concurrency theory, dating back at least to Winkowski [21].

CKA has both interleaving and true concurrency models, that is, shuffle as well as pomset languages. Series-parallel pomset languages, which are generated from singletons by finitary applications of sequential and parallel compositions, form free algebras in this class [10, 13] (at least when parallel iteration is ignored). The inherent compositionality of algebra is thus balanced by the generative properties of this model. Yet despite this and other theoretical work, applications of CKA remain rare.

One reason is that series-parallel pomsets are not expressive enough for many real-world applications: even simple producer-consumer examples cannot be modelled [14]. Tests, which are needed for the control structure of concurrent programs, and assertions are hard to capture in models of CKA (see [8] and its discussion in [9]). Finally, it remains unclear how modal operators could be defined over graph models akin to pomset languages, which is desirable for concurrent dynamic algebras and logics beyond alternating nondeterminism [3, 15].

A natural approach to generating more expressive pomset languages is to "cut across" pomsets in more general ways when (de)composing them. This can be achieved by (de)composing along interfaces, and this idea can be traced back again to Winkowski [21]. As a side effect, interfaces may yield notions of tests, assertions or modalities. When they consist of events, cutting across them presumes that they extend in time and thus form intervals. Interval orders [2, 20] of events with duration have been applied widely in partial order semantics of concurrent and distributed systems [6, 11, 12, 16–19] and the verification of weak memory models [4], yet generating them remains an open problem [7].

We propose a new class and algebra of posets with interfaces (*iposets*) based on these ideas. We introduce a new gluing composition that acts like standard serial po(m)set composition outside of interfaces, yet glues together interface events, thus composing events that did not end in one component with those that did not start in the other one. Our definitions are categorical so that isomorphism classes of posets are considered ab initio. We show that the hierarchy of gluing-parallel posets generated by finitary applications of this gluing composition and the standard parallel composition, starting from singleton iposets, contains both the series-parallel posets and the interval orders. Iposets thus retain the pleasant compositionality properties of series-parallel pomsets and the wide applicability of interval orders in concurrency and distributed computing.

This extended abstract is based on [1] which will be presented at the 18th International Conference on Relational and Algebraic Methods in Computer Science, RAMICS, in April 2020.

References

1. U. Fahrenberg, C. Johansen, G. Struth, and R. Bahadur Thapa. Generating posets beyond N. *CoRR*, abs/1910.06162, 2019.

2. P. C. Fishburn. Intransitive indifference with unequal indifference intervals. *J. Math. Psych.*, 7(1):144–149, 1970.
3. H. Furusawa and G. Struth. Concurrent dynamic algebra. *ACM Trans. Comput. Log.*, 16(4):30:1–30:38, 2015.
4. M. Herlihy and J. M. Wing. Linearizability: A correctness condition for concurrent objects. *ACM Trans. Program. Lang. Syst.*, 12(3):463–492, 1990.
5. T. Hoare, B. Möller, G. Struth, and I. Wehrman. Concurrent Kleene algebra and its foundations. *J. Log. Algebr. Program.*, 80(6):266–296, 2011.
6. R. Janicki and M. Koutny. Structure of concurrency. *Theor. Comput. Sci.*, 112(1):5–52, 1993.
7. R. Janicki and X. Yin. Modeling concurrency with interval traces. *Inf. Comput.*, 253:78–108, 2017.
8. P. Jipsen and M. A. Moshier. Concurrent Kleene algebra with tests and branching automata. *J. Log. Algebr. Meth. Program.*, 85(4):637–652, 2016.
9. T. Kappé, P. Brunet, J. Rot, A. Silva, J. Wagemaker, and F. Zanasi. Kleene algebra with observations. In *CONCUR 2019*, vol. 140 of *LIPIcs*. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2019.
10. T. Kappé, P. Brunet, A. Silva, and F. Zanasi. Concurrent Kleene algebra: Free model and completeness. In *ESOP 2018*, vol. 10801 of *LNCS*. Springer, 2018.
11. L. Lamport. The mutual exclusion problem: Part I - a theory of interprocess communication. *J. ACM*, 33(2):313–326, 1986.
12. L. Lamport. On interprocess communication. Part I: basic formalism. *Distributed Computing*, 1(2):77–85, 1986.
13. M. R. Laurence and G. Struth. Completeness theorems for pomset languages and concurrent Kleene algebras. *CoRR*, abs/1705.05896, 2017.
14. K. Lodaya and P. Weil. Series-parallel languages and the bounded-width property. *Theor. Comput. Sci.*, 237(1-2):347–380, 2000.
15. D. Peleg. Concurrent dynamic logic. *J. ACM*, 34(2):450–479, 1987.
16. R. J. van Glabbeek. The refinement theorem for ST-bisimulation semantics. In *IFIP TC2 Working Conf. Programming Concepts and Methods*. North-Holland, 1990.
17. R. J. van Glabbeek and F. W. Vaandrager. Petri net models for algebraic theories of concurrency. In *PARLE (2)*, vol. 259 of *LNCS*. Springer, 1987.
18. W. Vogler. Failures semantics based on interval semiwords is a congruence for refinement. *Distributed Computing*, 4:139–162, 1991.
19. W. Vogler. *Modular Construction and Partial Order Semantics of Petri Nets*, vol. 625 of *Lecture Notes in Computer Science*. Springer, 1992.
20. N. Wiener. A contribution to the theory of relative position. *Proc. Camb. Philos. Soc.*, 17:441–449, 1914.
21. J. Winkowski. An algebraic characterization of the behaviour of non-sequential systems. *Inf. Process. Lett.*, 6(4):105–109, 1977.