Approximate simulation and bisimulation relations for fuzzy automata

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Algorithms for checking and computing the behavioural equivalences of various systems have been investigated in many areas of mathematics and computer science under different names, most commonly as simulations and bisimulations. They have also taken an important place in the automata theory, or more precisely, in the context of nondeterministic [2, 5], weighted [1, 4], fuzzy [3] and probabilistic automata, etc.

Roughly speaking, two states of fuzzy automata are considered bisimilar if and only if they can perform the same actions to reach bisimilar states. As it turns out, this condition can be too rigorous even for some simple real time systems. The study of approximate bisimulation relations for fuzzy automata has lately received a wide attention [6, 7, 8, 9, 10].

In this talk, we introduce two types of λ-approximate simulations and four types of λ-approximate bisimulations for fuzzy automata, where λ is the degree taken from the underlying set of truth values, which is a complete Heyting algebra. We show that λ-approximate simulations between fuzzy automata imply that the degree of inclusion of the corresponding fuzzy languages is at least λ, while λ-approximate bisimulations imply that the degree of equality of the corresponding fuzzy languages is at least than λ. Moreover, we show that, for any λ-approximate (bi)simulation, the exact degree of fuzzy language similarity (or equality) can be computed. We investigate further properties of λ-approximate (bi)simulations, and in particular, we pay attention to those λ-approximate (bi)simulation which are uniform fuzzy relations. In the end, we show that λ-approximate (bi)simulations are superior in the state reduction of fuzzy automata when compared to exact (bi)simulations, in the sense that there are cases when no reduction can be made using fuzzy (bi)simulations, while it is possible to reduce the number of states of a given fuzzy automaton using λ-approximate (bi)simulations.

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