Weakly linear systems for matrices over the max-plus quantale and their applications

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Weakly linear systems are a special kind of systems of relational or matrix inequations and equations that have recently emerged in the study of fuzzy automata, and have also been applied in social network analysis. They have been used to reduce the number of states of fuzzy automata [9–12], in the study of simulations and bisimulations for nondeterministic, fuzzy and weighted automata [1–4], as well as in the positional analysis of social networks [5, 6]. In addition, certain types of WLS have been used to improve determinization algorithms for fuzzy automata [8] and in the conflict analysis of fuzzy discrete event systems [12]. In all these cases, the underlying structure of membership values was a complete residuated lattice.

General properties of weakly linear systems of fuzzy relation inequations and equations, with a complete residuated lattice as the underlying structure of membership values, have been studied in [5–7]. The key role in this study is played by completeness and residuation in the structure of membership values, which ensure completeness and residuation in the corresponding lattices of fuzzy relations. The residuation reduces the problem of solving WLS to the problem of computing post-fixed points of suitable isotone functions on the lattice of fuzzy relations, and the completeness of this lattice enables to use the Knaster-Tarski Fixed Point Theorem, according to which solutions of a WLS form a complete lattice, and therefore, there exists the greatest solution. The greatest solutions of WLS are computed using a procedure which is a modification of the Kleene Fixed Point Theorem. This procedure does not necessarily terminate in a finite number of steps, and some sufficient conditions have been found when this happens.

Here we consider weakly linear systems in the context of matrices over the max-plus algebra. The complete max-plus algebra is very similar to complete residuated lattices[1], the only difference is that its multiplicative identity is not the greatest element. In fact, the complete max-plus algebra is a commutative unital quantale. We prove the existence of the greatest solution contained in a given matrix X_0, and present a procedure for its computation. In case of finite WLS when all finite entries of matrices forming this WLS and the matrix X_0 are integers, rationals or particular irrationals and a finite solution exists, the procedure finishes in a finite number of steps. If in that case an arbitrary finite

[1] we are talking about complete residuated lattices defined in the way that is the most common in the fuzzy set theory (cf. [2, 3, 10, 5–8, 11, 12]).

solution is given, a lower bound on number of computational steps is calculated. Otherwise, we use our algorithm to compute approximations to finite solutions.

We will also mention applications of WLS in the study of max-plus automata.

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