

Weighted context-free grammars over a complete strong bimonoid

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Weighted context-free grammars are classical context-free grammars in which the productions carry weights. Adding weights allows a quantitative view of derivations, which can be very useful in many applications. Traditionally, the weights are assumed to form the algebraic structure of a semiring, but recently other ways of weight computation are also considered. Shang, Lu and Lu [8] studied context-free grammars with weights taken from particular algebraic structures called lattice-ordered quantum MV algebras. Droste and Vogler [3] were dealing with CFGs over unital valuation monoids, whereas Jin and Li [5] and Rahonis and Torpari [7] have considered CFGs over bimonoids.

Here we are interested in weighted context-free grammars over complete strong bimonoids. Due to lack of distributivity we distinguish between two ways to define behaviour of such grammars: *depth-first semantics* and *breadth-first semantics*. In the depth-first semantics, we associate the weight of the derived word by summing weights of all derivations of that word, where the weight of a derivation is computed multiplying weights of the productions that form it. Contrary to that, in the breadth-first semantics, we compute weights of derivations of different lengths, gradually increasing the length. If the underlying weight-structure is a semiring, we prove that these two semantics coincide. Moreover, we show that two semantics coincide for every weighted context-free grammar G over K without multiple direct derivations if and only if the underlying complete strong bimonoid K is right distributive. When K is an additively idempotent complete strong bimonoid, we prove that two semantics coincide for every weighted context-free grammar G over K if and only if K is a semiring.

Restricting attention only to grammars with leftmost derivations, we define *leftmost depth-first semantics* and *leftmost breadth-first semantics* and we show that these two semantics coincide for every weighted context-free grammar G over K if and only if K is right distributive.

Similar issues will be considered in the context of weighted pushdown automata, along with the question of the equivalence of weighted context-free grammars and weighted pushdown automata.

References

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