Kleene and Büchi for Weighted Forest Languages over M-Monoids

Frederic Dörband[1]
Technische Universität Dresden, February 10, 2020

We recall forests as finite tuples of trees. This syntactic extension of trees already occurred in [2,3,4]. We define weighted forest automata (wfa) as an extension of weighted tree automata (wta). Our definition generalizes the unweighted forest automata introduced in [1] to the weighted case, while also generalizing the weighted forest automata introduced in [3] from commutative semirings to M-monoids. In comparison with [2] , our paper is a generalization from semirings to M-monoids, but also a restriction from hedge languages to forest languages.

A wfa is syntactically similar to a wta, however the semantics of wfa allow for entire forests to be processed (instead of single trees). The essence of our paper is a theorem stating that our automaton model generates only so-called rectangular weighted forest languages. That is, for every recognizable weighted forest language φ, there are recognizable weighted tree languages (the so-called rectangular components of φ), whose horizontal concatenation equals φ.

Using this rectangularity property, we then infer both, a Kleene-like result and a Büchi-like result for weighted forest languages. On the one hand, this generalizes the Kleene-like result for weighted tree automata from [5] to the case of forests and the Kleene-like result for unweighted forest automata [1] to the case of M-monoids. On the other hand, this generalizes the Büchi-like result for weighted tree automata from [6] to the case of forests.

For our Kleene-like result, we introduce rational forest expressions, which are horizontal products of rational tree expressions and for our Büchi-like result, we introduce forest M-expressions, which are horizontal products of tree M-expressions. We ultimately ask the question, whether the definitions for rational forest expressions and forest M-expressions can be done inductively. However, we see that straightforward inductive definitions are not possible due to ambiguity in the choice of rectangular components.

[1]Research of this author is supported by the DFG Research Training Group 1763 “QuantLA”

[2]L. Straßburger. “A Kleene theorem for forest languages”. In: *Language and automata theory and applications.* Vol. 5457. Lecture Notes in Comput. Sci. Springer, Berlin, 2009, pp. 715–727. URL: `https://doi.org/10.1007/978-3-642-00982-2_61`.

[3]C. Mathissen. *Weighted Automata and Weighted Logics over Tree-like Structures.* Citeseer, 2009.

[4]F. Dörband. “A Kleene Theorem for Weighted Forest Automata”. MA thesis. Technische Universität Dresden, 2019.

[5]Z. Fülöp, A. Maletti, and H. Vogler. “A Kleene theorem for weighted tree automata over distributive multioperator monoids”. In: *Theory Comput. Syst.* 44 (2009), pp. 455–499.

[6]Zoltán Fülöp, Torsten Stüber, and Heiko Vogler. “A Büchi-like theorem for weighted tree automata over multioperator monoids”. In: *Theory of Computing Systems* 50.2 (2012), pp. 241–278.