Coevolution of habitat use in stochastic environments

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In collaboration with Alex Hening (Tufts) and Dang Nguyen (University of Alabama)

February 13, 2020

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A suggestion came from someone who wrote

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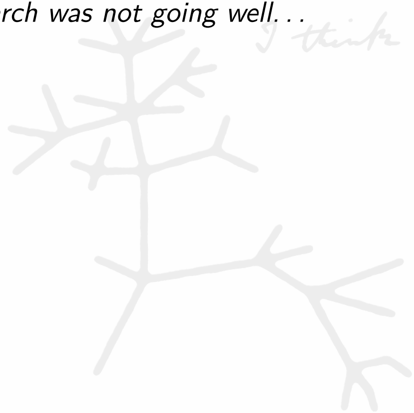

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CC/NUMBER 8
FEBRUARY 25, 1991

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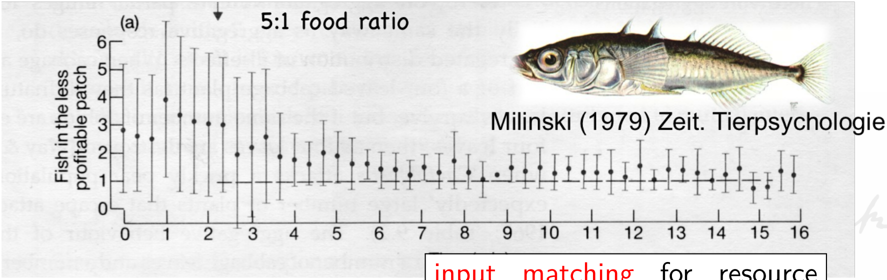
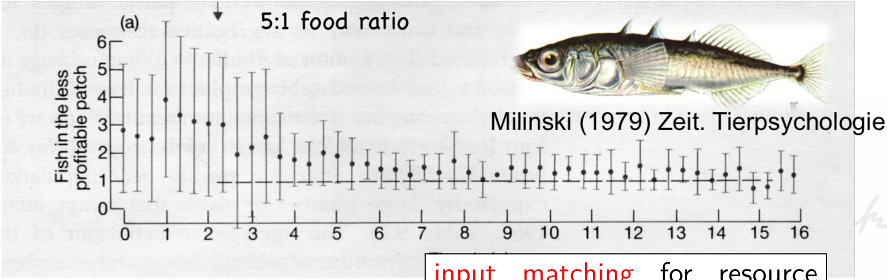
Can lead to...



input matching for resource consumption [Parker, 1978]



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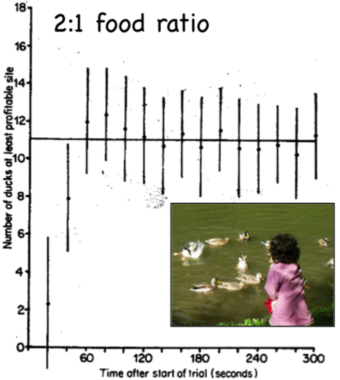
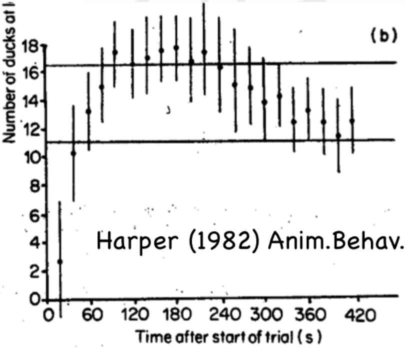
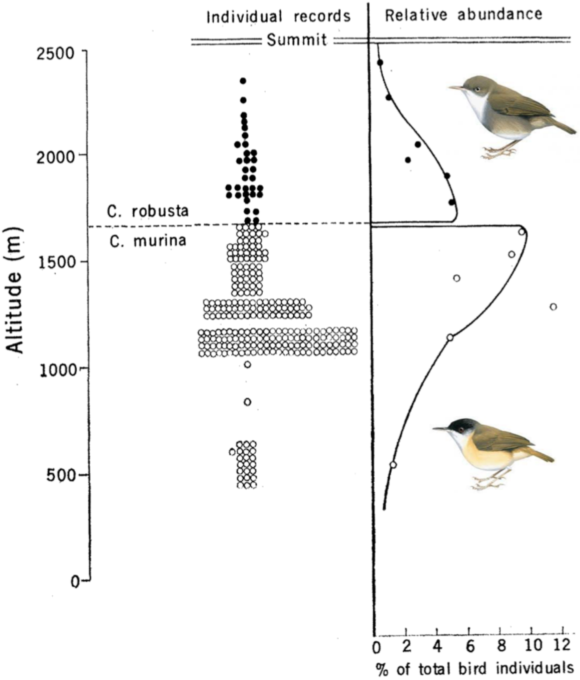
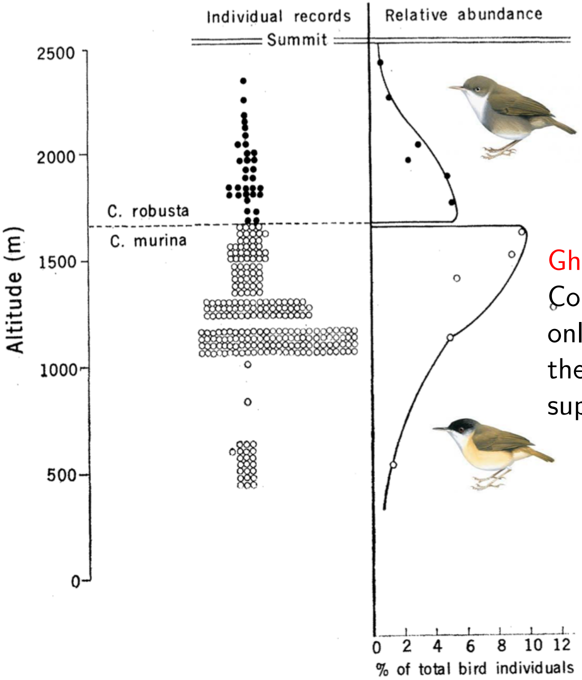




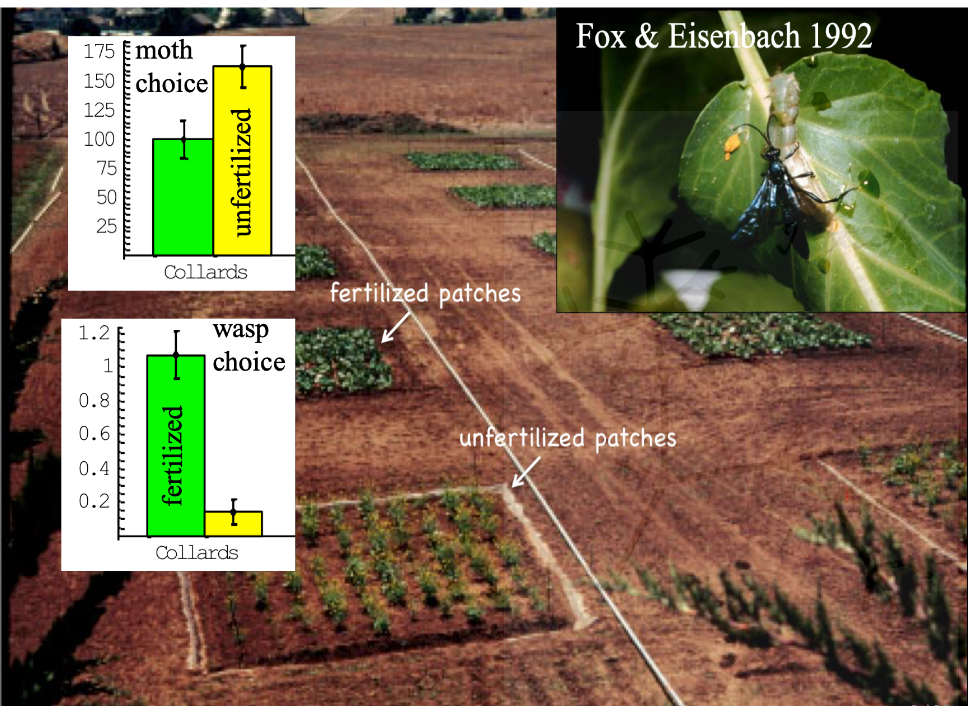


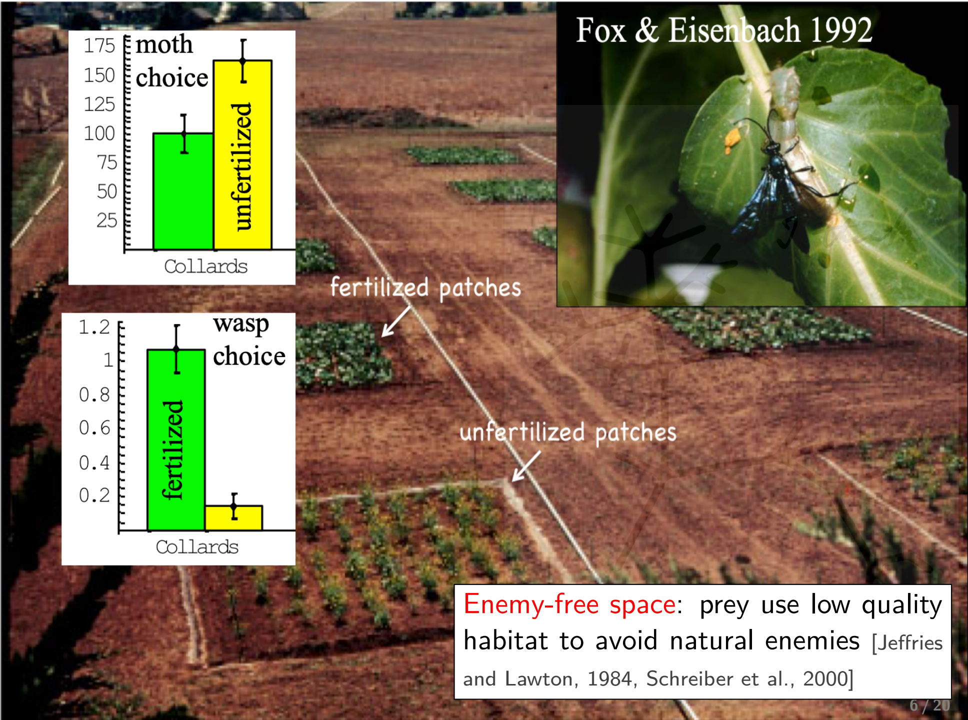
Individual records
Relative abundance
Summit
2500
2000
1500
1000
500
0
Altitude (m)
C. robusta
C. murina
0 2 4 6 8 10 12
% of total bird individuals



Ghost of competition past: Competing species evolve to only select habitats where they are competitively superior. Connell [1980]

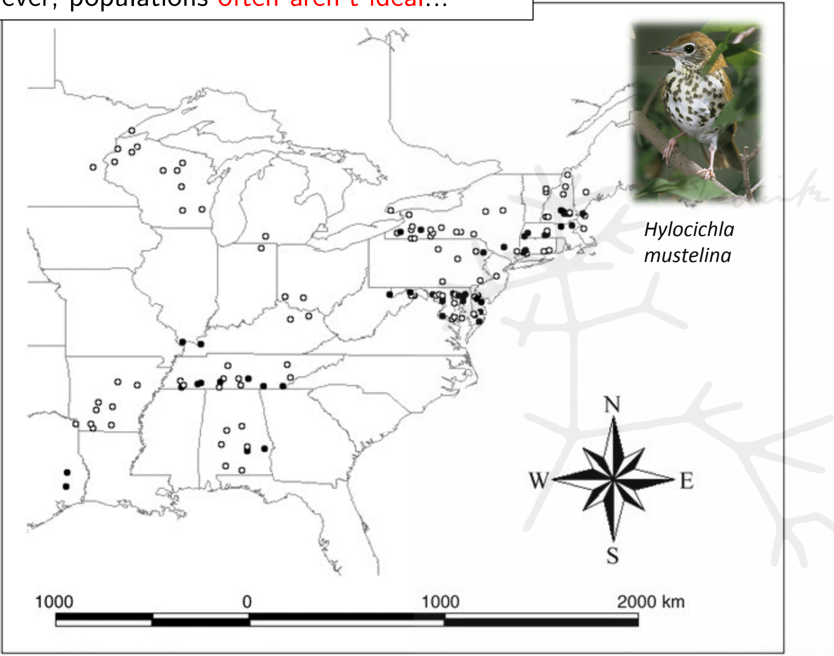

Fox & Eisenbach 1992
fertilized patches
unfertilized patches


Fox & Eisenbach 1992
moth choice
175
150
125
100
75
50
25
unfertilized
Collards
wasp choice
1.2
1
0.8
0.6
0.4
0.2
fertilized
Collards
fertilized patches
unfertilized patches



Enemy-free space: prey use low quality habitat to avoid natural enemies [Jeffries and Lawton, 1984, Schreiber et al., 2000]

However, populations often aren't ideal...



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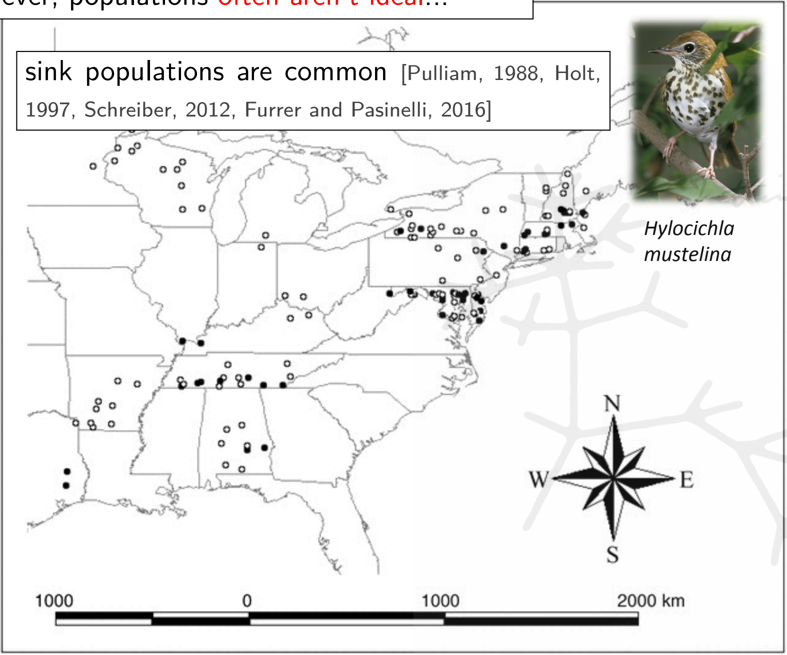
sink populations are common [Pulliam, 1988, Holt, 1997, Schreiber, 2012, Furrer and Pasinelli, 2016]



Hylocichla mustelina

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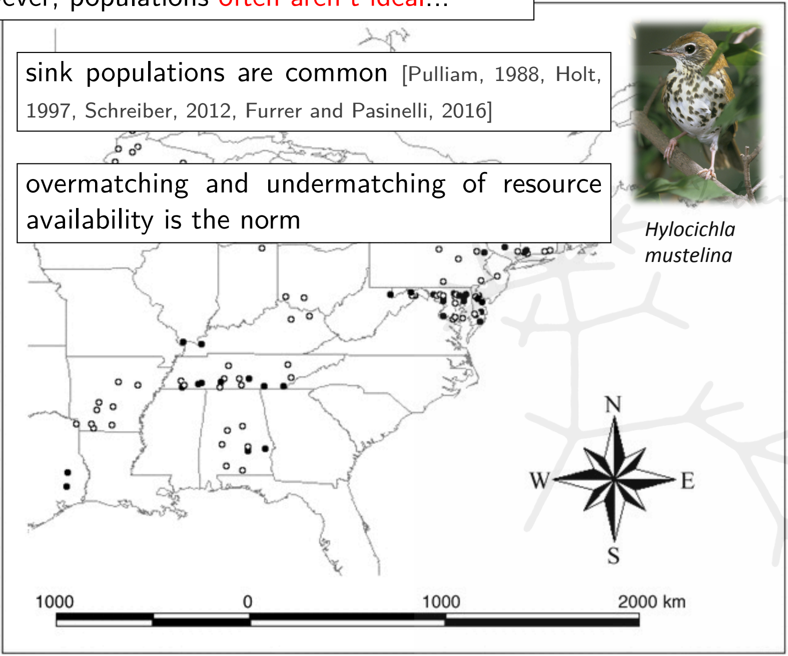

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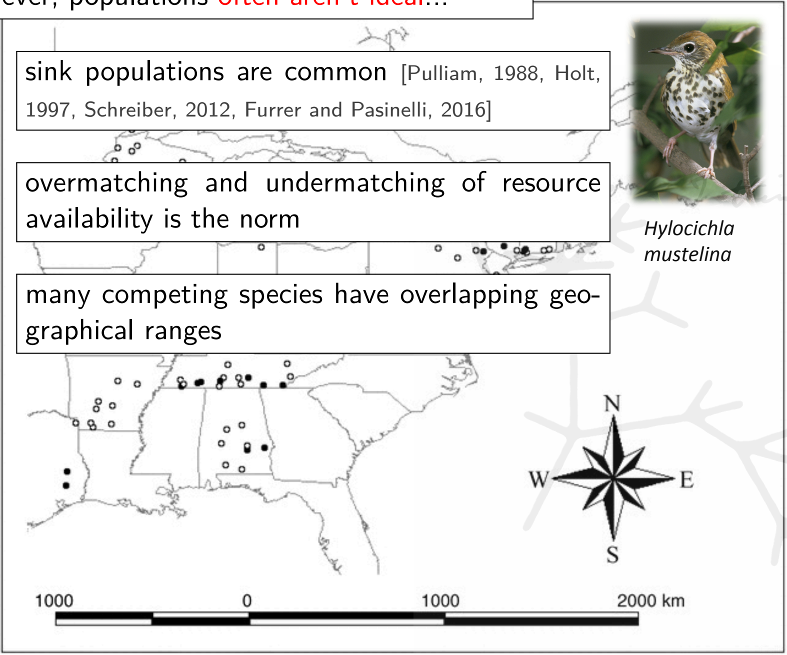

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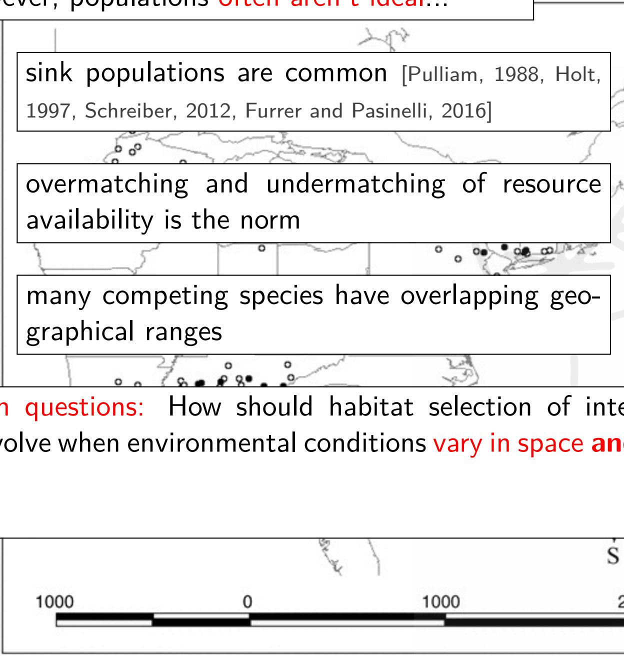

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Main questions: How should habitat selection of interacting species coevolve when environmental conditions vary in space **and time**?

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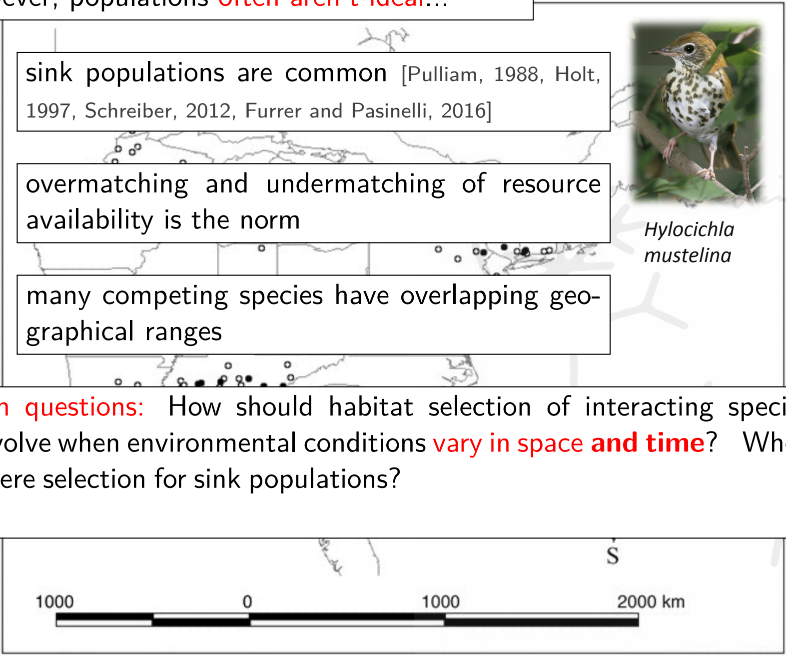

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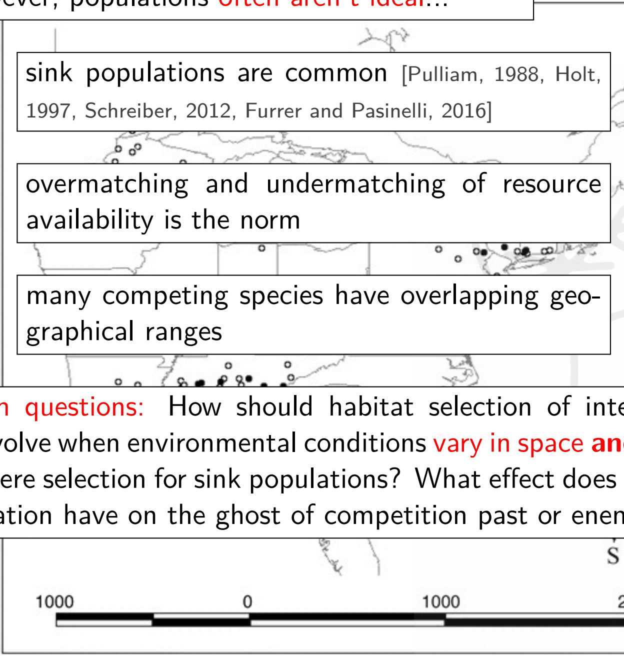
Main questions: How should habitat selection of interacting species coevolve when environmental conditions vary in space **and time**? When is there selection for sink populations? What effect does spatio-temporal variation have on the ghost of competition past or enemy-free space?




KISS
ROCK AND ROLL OVER
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Implicit space!

Implicit space!
Mass action!!

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Mass action!!
Diffusion approximations!!!

Dynamics within patch ℓ

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In limit $\Delta t \downarrow 0$, get the Itô stochastic differential equations (SDEs)

$$dx_i^\ell(t) = x_i^\ell(t) \left(\left(\sum_{j=1}^{n} a_{ij}^\ell x_j^\ell(t) + b_i^\ell \right) dt + dE_i^\ell(t) \right)$$

where $E_i^\ell(t)$ is a Brownian motion with mean 0 and variance $\sigma_i^{\ell\ell} t$

Spatial coupling of patch dynamics

Assume

$$\mathrm{Cov}[\mathrm{x}_\mathrm{i}^\ell(\mathrm{t}+\Delta\mathrm{t})-\mathrm{x}_\mathrm{i}^\ell(\mathrm{t}),\mathrm{x}_\mathrm{i}^\mathrm{m}(\mathrm{t}+\Delta\mathrm{t})-\mathrm{x}_\mathrm{i}^\mathrm{m}(\mathrm{t})\mid\mathrm{x}(\mathrm{t})]\approx\mathrm{x}_\mathrm{i}^\ell(\mathrm{t})\mathrm{x}_\mathrm{i}^\mathrm{m}(\mathrm{t})\sigma_\mathrm{i}^{\ell\mathrm{m}}\Delta\mathrm{t}$$

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where $B_i(t)$ are Brownian motions satisfying $\mathrm{Var}[B_i(t)] = t$


I think

To persist or not to persist, that is the question

Consider

$$dx_i(t) = x_i(t)\left[f_i(x(t))dt + \sqrt{V_i(x(t))}dE_i(t)\right] \qquad (\bigstar)$$

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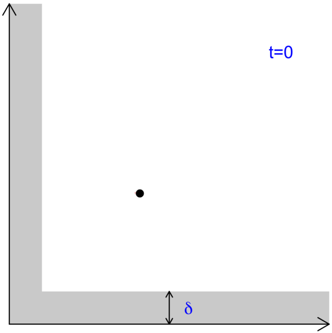

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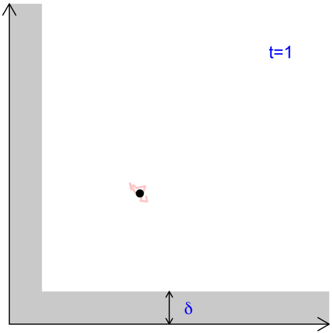

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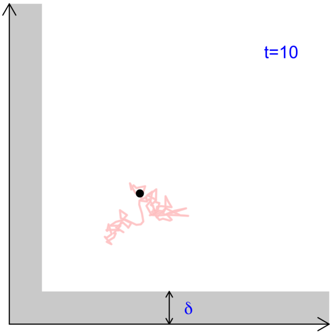

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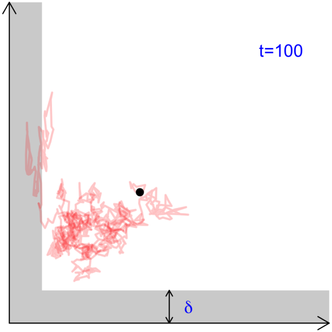

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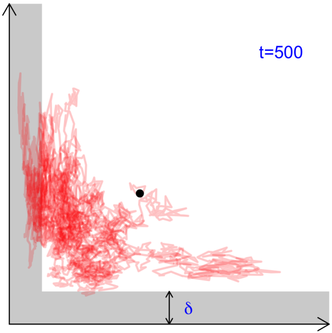

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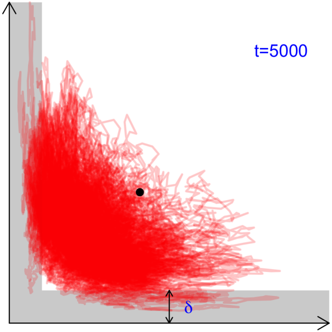

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Hening and Nguyen [2018], Benaïm [2018] extended $(\clubsuit)$ to non-compact domains (e.g. LV system) with additional condition to ensure tightness

Resident-Mutant dynamics

$x(t) = (x_1(t), \ldots, x_n(t))$ w/ p_i^{ℓ}

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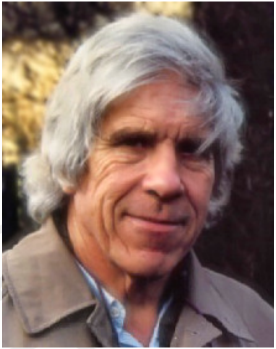
unbeatable strategy *[Hamilton, 1967]* *"This word was applied in just the same sense in which it could be applied to the 'minimax' strategy of a zero-sum two-person game.*



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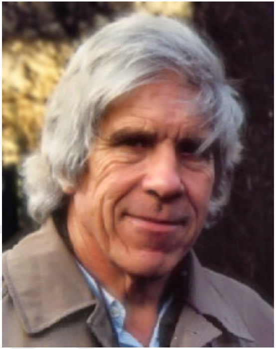

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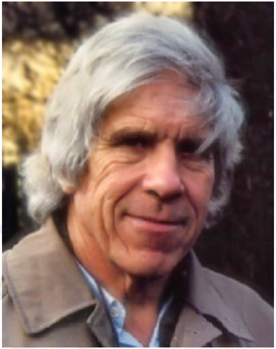
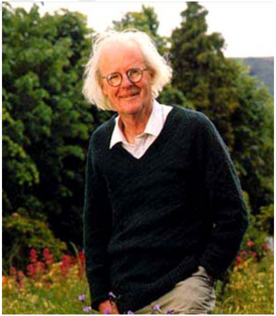


Evolutionarily stable strategy *[Smith and Price, 1973]* *- a strategy that cannot be invaded by any other strategy that is initially rare*



Invasion rates $\mathcal{I}(p, q_i) := \sum_{\ell} q_i^{\ell} \overbrace{\left(\sum_{j=1}^{n} a_{ij}^{\ell} p_j^{\ell} \mathbb{E}[\widehat{x}_j] + b_i^{\ell} \right)}^{=: f_i^{\ell}(p)} - \frac{1}{2} \overbrace{\sum_{j,\ell} q_i^{\ell} q_i^{m} \sigma_i^{\ell m}}^{=: V_i(q)}$

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perfectly correlated fluctuations $\Rightarrow$ ideal free distribution

imperfectly correlated fluctuations $\Rightarrow$ local growth rate $f_i^\ell(p) - \frac{\sigma_i^{\ell\ell}}{2} < 0$ in all occupied patches!!!!

Application: Competing species

$$dx_i^\ell(t) = x_i^\ell(t)\left[\left(r_i^\ell - x_1^\ell(t) - x_2^\ell(t)\right)dt + dE_i^\ell(t)\right] \qquad i = 1, 2$$

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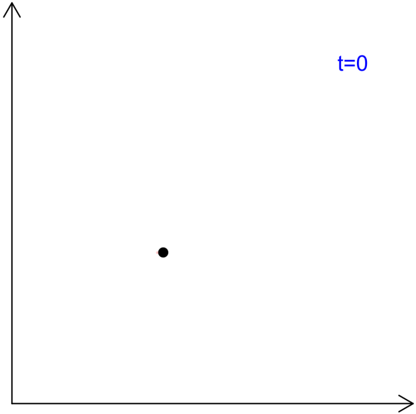

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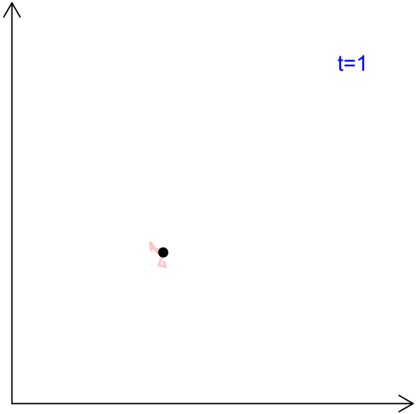

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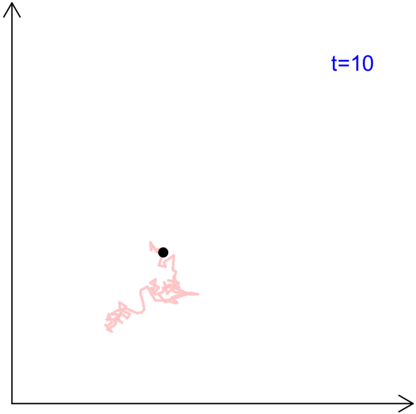

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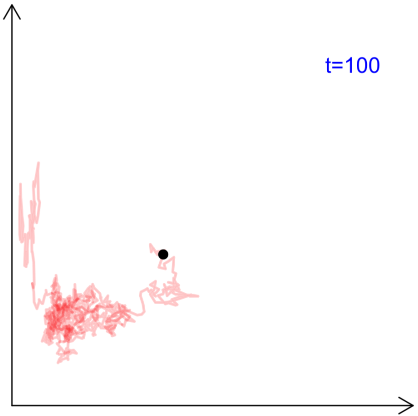

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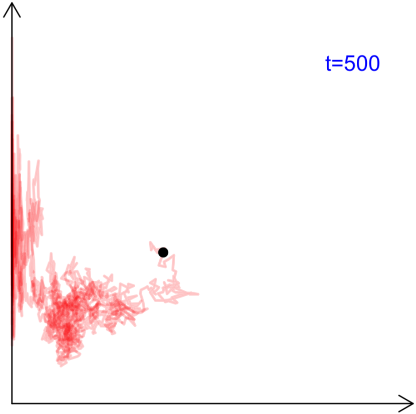

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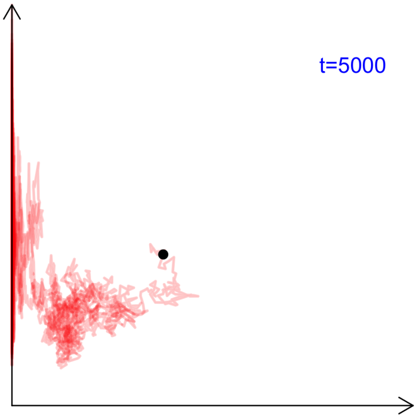

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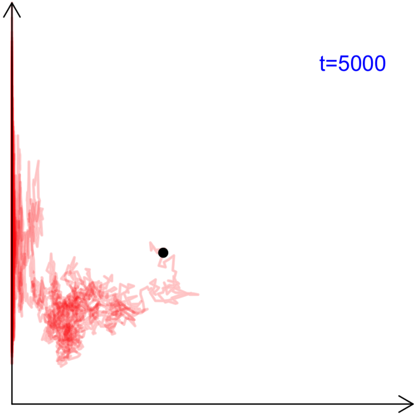
Now, spatially couple patches with $X_i^\ell = p_i^\ell X_i$...



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$$dx_i(t) = x_i(t) \sum_{\ell} p_i^{\ell} \left[\left(r_i^{\ell} - p_1^{\ell} x_1(t) - p_2^{\ell} x_2(t) \right) dt + dE_i^{\ell}(t) \right] \qquad i = 1, 2$$

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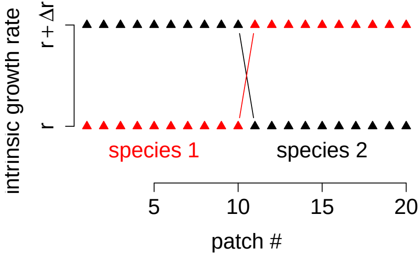

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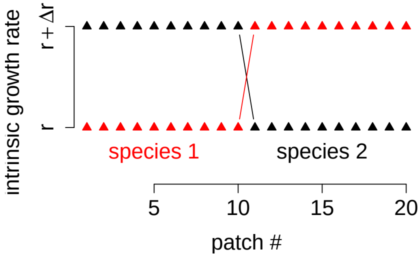

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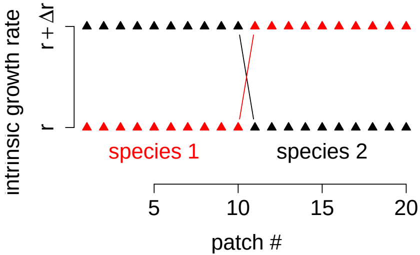

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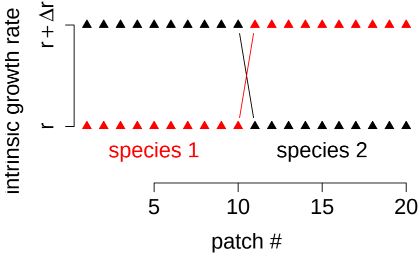

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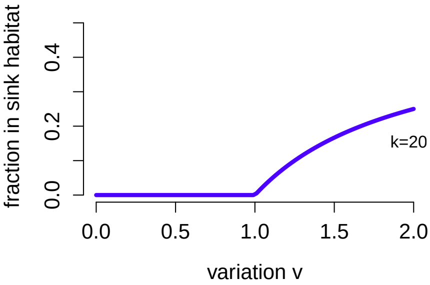

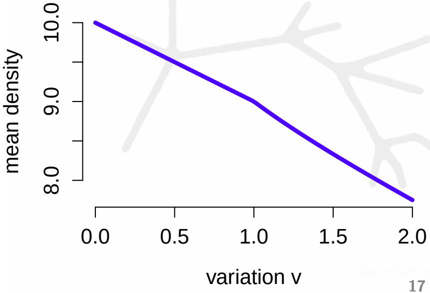

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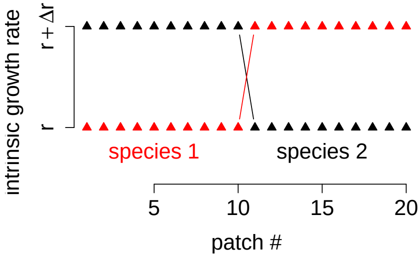

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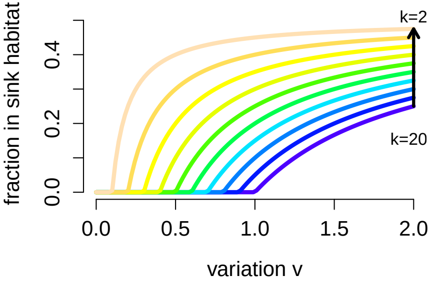

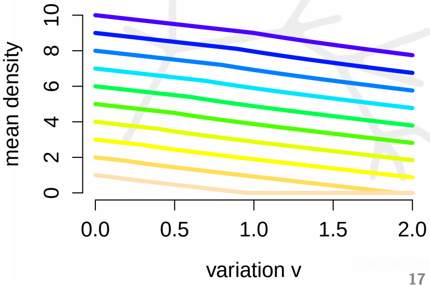

Application: Predator-prey interactions

prey $$dx_1(t) = x_1(t) \sum_{\ell} p_1^{\ell} \left[\left(r^{\ell} - \varepsilon p_1^{\ell} x_1 - a p_2^{\ell} x_2(t) \right) dt + dE_1^{\ell}(t) \right]$$

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- $v^{***} < v \Rightarrow$ predator only uses sink habitat i.e. $p_1^2 > 0$, $p_2^2 = 0$

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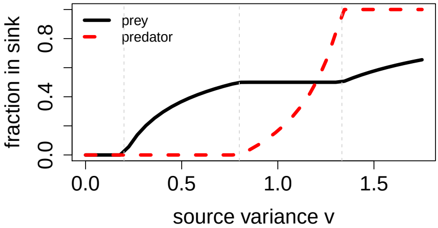

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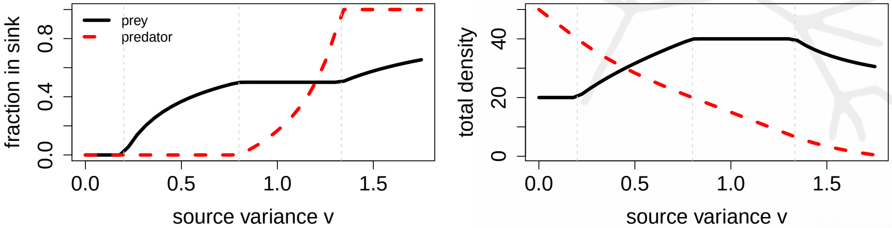

Finale

Perfectly correlated fluctuations across space select for an ideal free distribution whereby local per-capita growth rates $= 0$ in occupied patches, < 0 elsewhere

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You for listening! Questions?

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