$D(n)$-sets with square elements

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For an integer n, a set of distinct nonzero integers $\{a_1, a_2, ..., a_m\}$ such that $a_i a_j + n$ is a perfect square for all $1 \leq i < j \leq m$, is called a Diophantine m-tuple with the property $D(n)$ or simply a $D(n)$-set. $D(1)$-sets are known as Diophantine m-tuples. When considering $D(n)$-sets, usually an integer n is fixed in advance. However, we may ask if a set can have the property $D(n)$ for several different n's. For example, $\{8, 21, 55\}$ is a $D(1)$-triple and $D(4321)$-triple. In a joint work with Adžaga, Kreso and Tadić, we presented several families of Diophantine triples which are $D(n)$-sets for two distinct n's with $n \neq 1$. In a joint work with Petričević we proved that there are infinitely many (essentially different) quadruples which are simultaneously $D(n_1)$-quadruples and $D(n_2)$-quadruples with $n_1 \neq n_2$. Morever, the elements in some of these quadruples are squares, so they are also $D(0)$-quadruples. E.g. $\{54^2, 100^2, 168^2, 364^2\}$ is a $D(8190^2)$, $D(40320^2)$ and $D(0)$-quadruple. In this talk, we will describe methods used in constructions of mentioned triples and quadruples. We will also mention a work in progress with Kazalicki and Petričević on $D(n)$-quintuples with square elements (so they are also $D(0)$-quintuples). There are infinitely many such quintuples. One example is a $D(480480^2)$-quintuple $\{225^2, 286^2, 819^2, 1408^2, 2548^2\}$.