Maximal highly proximal flows of locally compact groups

The notion of a highly proximal extension of a flow generalizes the one of an almost one-to-one extension (injective on a dense G_δ set), which is an important tool in topological dynamics. The existence of maximal such extensions was proved by Auslander and Glasner in the 70s for minimal flows using an abstract argument, and a concrete construction using near-ultrafilters was recently given by Zucker for arbitrary flows. When the acting group is discrete, the MHP extension is nothing but the Stone space of the Boolean algebra of the regular open sets of the space. We give yet another construction of the MHP extension for arbitrary topological groups and prove that for MHP flows of a locally compact group G, the stabilizer map $x \to G_x$ is continuous (for general flows, this map is only semi-continuous). This is a common generalization of a theorem of Frolík that the set of fixed points of a homeomorphism of a compact, extremally disconnected space is open and a theorem of Veech that the action of a locally compact group on its greatest ambit is free. This is joint work with Adrien Le Boudec.