Commability and graphs of abelian groups

We consider the class of finitely generated groups admitting a non-elementary and cocompact action on a locally finite tree, with vertex stabilizers virtually free abelian of rank $n \geq 1$. In the case $n=1$, Baumslag-Solitar groups are examples of such groups. The behavior of that class of groups with respect to quasi-isometries is described by works of Mosher-Sageev-Whyte, Farb-Mosher, and Whyte.

We study the commability rigidity problem for this class of groups. Such a group Γ admits a canonical linear representation ρ_Γ over an n-dimensional Q-vector space. Our main result provides a necessary criterion for two groups Γ,Λ to be commable, in terms of the images of the representations ρ_Γ and ρ_Λ. This result complements Whyte's quasi-isometric classification within this class of groups, and it implies that many groups in this class are not commable, although they are quasi-isometric.

Joint work with Yves Cornulier.