Decomposition and construction of cubic and non-cubic neighbourhood operations

Atsushi Imiya*[1]

[1]IMIT Chiba University – Japan

Abstract

We introduce a method to construct morphological operations in a higher-dimensional digital space from a collection of set operations in lower dimensional digital spaces. Together with decomposition of digital objects, the decomposition of the neighbourhood shows that the neighbourhood-based operation in the higher-dimensional digital spaces can be decomposed into the union of neighbourhood-based operations the in lower-dimensional digital spaces.
First, we prove that the 2n-neighbourhood in an n-dimensional digital space is decomposed into the 2(n-1)-neighbourhoods in the mutually orthogonal (n-1)-dimensional digital spaces. This decomposition and construction relation of the neighbourhoods and objects implies that morphological operations in an n-dimensional digital space can be computed as the union of one- and two-dimensional morphological operations on isothetic digital lines and planes intersecting with the digital object in the digital space. These decomposition properties are applied for the boundary detection and medial-set extraction.

Both in two- and three-dimensional digital space, there are non-square and non-cubic grid systems for digital tessellation of space. Hexagonal grid system is a digital tessellation on a plane derived by hexagonal tiling. In three-dimensional space, spacing by rhombic dodecahedron derives a grid system for digital volumetric data. This tessellation is called FCC-grid system.

Second, we extend these decomposition properties to non-cubic grid systems. The six-neighbourhood in digital space is decomposed to three four-neighbourhoods on mutually orthogonal planar digital plane. This decomposition property of neighbourhood allows us to construct digital operation in three-dimensional digital space from these on digital planes. Furthermore, planar four-neighbourhood is decomposed two two-orthogonal neighbourhood along mutually orthogonal lines.

We derive neighbourhood decomposition in FCC-grid system, that is, the FCC-grid system is decomposed into four hexagonal grid systems. Each hexagonal system is orthogonal to the rotation axis of the tetrahedron, which passes through the centroid and a vertex of the tetrahedron.

The six-neighbourhood of hexagonal grid system is decomposed to three two-neighbourhoods along lines whose crossing angles are 120 degrees. These decomposition properties of rhombic dodecahedral and hexagonal grid-systems allow us to construct digital operations in

*Speaker

non-cubic grid systems as combinations of operations along lines.

Planar six-neighbourhood for hexagonal grid system is derived as the projection of cubic-grid system onto the plane along the longest diagonal of cube.
We extend these decomposition properties to non-cubic grid by using the orthogonal projection of four-dimensional eight-neighbourhood into three-dimensional space. Then, we derive relations between neighbourhoods in cubic and non-cubic grid systems.