R. Yin, K. Ziegler, F. Thiel and E. Barkai,
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Large fluctuations of the first detected quantum return time

Ruoyu Yin
Bar Ilan University

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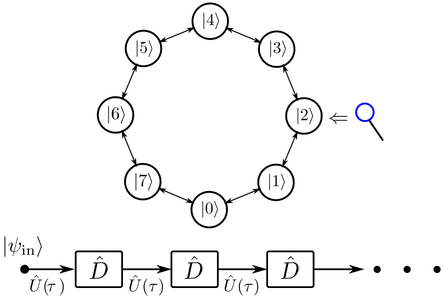
Repeated strong measurements until the first success:



$$\hat{\mathrm{U}}(\tau) = \mathsf{exp}(-\mathrm{iH}\tau)\ (\hbar = 1),\ \hat{\mathrm{D}} = |\psi_{\mathrm{d}}\rangle\langle\psi_{\mathrm{d}}|$$

Outcome: a string of length n: "$0, 0, 0, \ldots, 1$" ("0" for non-detection, and "1" for detection)

If

- First-detected "return" time (FDRt): $|\psi_{\rm d}\rangle = |\psi_{\rm in}\rangle$
- The system's energy spectrum is discrete

Known results: (Grünbaum, Velázquez et al. 2013)

- The FDR amplitude, the generating function(Z Transform):
$\phi_{\rm n} = \langle\psi_{\rm in}| \left[\hat{U}(\tau)(1-\hat{D})\right]^{\rm n-1} \hat{U}(\tau)\,|\psi_{\rm in}\rangle\,,\ \tilde{\phi}(z) = \frac{\langle\psi_{\rm in}|\sum_{n=1}^{\infty} z^n \hat{U}(n\tau)|\psi_{\rm in}\rangle}{1+\langle\psi_{\rm in}|\sum_{n=1}^{\infty} z^n \hat{U}(n\tau)|\psi_{\rm in}\rangle}$
- The system is recurrent
- The mean: $\langle n\rangle = \sum_{n=1}^{\infty} n|\phi_n|^2 = w \in \mathbb{Z}$, = the number of zeros $\{z_i\}$ of $\tilde{\phi}(z)$ [or the phases $\exp(iE_k\tau)$] =winding number of $\tilde{\phi}(e^{i\theta})$
- The variance:

$$\mathrm{Var}(n) = \left\langle n^2 \right\rangle - \langle n\rangle^2 = \sum_{i,j=1}^{w} \frac{2 z_i z_j^*}{1 - z_i z_j^*}$$

- Charge Theory: $\{z_i\}$ (excluding $z_0 = 0$)

$$V(z) = \sum_{k=1}^{w} p_k \ln|e^{iE_k\tau} - z| \Rightarrow \text{stationary points}$$

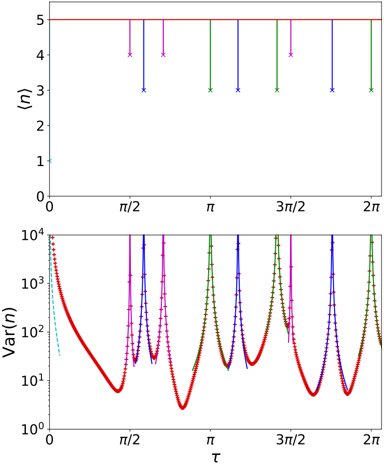
$$F(z) = \sum_{k=1}^{w} \frac{p_k}{e^{iE_k\tau} - z} \Rightarrow \text{equilibrium points}$$

where $p_k = \sum_{l=1}^{g_k} |\langle\psi_{\rm in}|E_{kl}\rangle|^2 \Longrightarrow$ charges on the unit circle.

Example

$$\mathrm{H} = -\gamma \sum_{\mathrm{x}=0}^{7} \left[|\mathrm{x}\rangle\langle \mathrm{x}+1| + |\mathrm{x}+1\rangle\langle \mathrm{x}| - 2|\mathrm{x}\rangle\langle \mathrm{x}| \right].$$

$$\mathrm{E_k}/\gamma = 2 - 2\cos(\pi \mathrm{k}/4)$$



Schematics

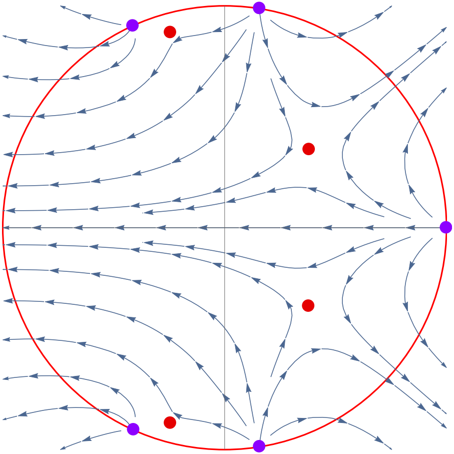


Figure: Eight-site ring. $\tau = 1$.

Mechanism of Jumps of $\langle n \rangle$ and Diverging Var(n)

Some zero(s) approach the unit circle:

- Mean: when the zero(s) "reach" the unit circle, it(they) does(do) not count into $\langle n \rangle$ (canceled out)
- Var(n): when the zero(s) is(are) approaching to the unit circle $\Longrightarrow$ vanishing denominator $(1 - z_i z_j^*)$ $\Longrightarrow$ diverging var(n)
  "Signature" to jumps of the topological number

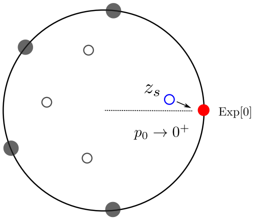
From basic knowledge of electrostatics:

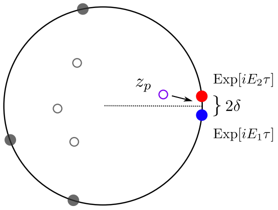
- some "weak" charge(s) ("Shrinking", Grünbaum, Velázquez et al. 2013): L_1 point in sun-earth system
- charges merging ("Fusion", Grünbaum, Velázquez et al. 2013): neighboring stars

Perturbation Method

$$F(z) = \sum_{k=1}^{w} \frac{p_k}{e^{iE_k\tau} - z} = 0 \Rightarrow \text{critical zero } \; z_c$$

$$\mathrm{Var}(n) \sim \frac{2|z_c|^2}{1 - |z_c|^2}$$





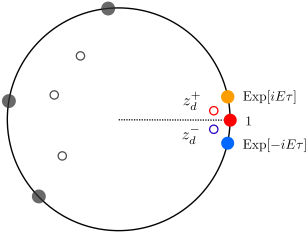
$$z_s \sim 1 - \frac{p_0}{\sum_{j\neq 0} p_j/[1-\exp(iE_j\tau)]}$$

$$z_p \sim 1 + i\frac{p_1-p_2}{p_1+p_2}\delta + \left[\frac{4p_1p_2}{(p_1+p_2)^3}\sum_{j\neq 1,2}\frac{p_j}{e^{iE_j\tau}-1} - \frac{1}{2}\right]\delta^2$$

$$\mathrm{Var}(n) \sim \frac{1}{2p_0}\left\{1 + \left[\sum_{j\neq 0} p_j \cot\left[(E_j - E_0)\,\tau/2\right]\right]^2\right\}$$

$$\mathrm{Var}(n) \sim 2\frac{(p_1+p_2)^3}{p_1p_2}\frac{1}{\tau^2\left(\bar{E}_2-\bar{E}_1\right)^2}$$

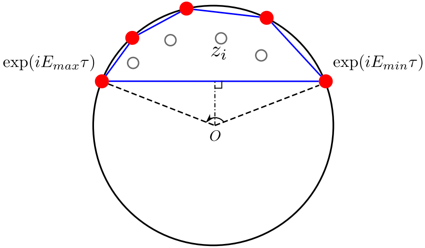
Triple-Charge Theory: Symmetric Scenario



$$\mathrm{Var(n)} \sim \sum_{\sigma=\pm} \frac{2|\mathrm{z}_\mathrm{d}^\sigma|^2}{1-|\mathrm{z}_\mathrm{d}^\sigma|^2} + \underbrace{\left(\frac{2\mathrm{z}_\mathrm{d}^+(\mathrm{z}_\mathrm{d}^-)^*}{1-\mathrm{z}_\mathrm{d}^+(\mathrm{z}_\mathrm{d}^-)^*} + \mathrm{c.c.}\right)}_{\mathcal{M}},$$

$$\mathrm{Var(n)} \sim 16\frac{(\mathrm{p}_0+2\mathrm{p})^2}{\mathrm{p}}\frac{1}{\tau^2\left(\bar{\mathrm{E}}_+-\bar{\mathrm{E}}_-\right)^2}, \text{the mixing term } \mathcal{M} \text{ is finite.}$$

Zeno Regime–Lower Bound



$$\mathrm{Var(n)} \geq (\mathrm{w}-1)\left[2\cot^2\left(\Delta \mathrm{E_m}\tau/2\right) - \mathrm{w} + 2\right]$$

Time-Energy Uncertainty Principle

Assume $\Delta E_m \tau \ll \hbar$

$$(\Delta E_m)^2 (\Delta t_{det})^2 \gtrsim 8(\langle n \rangle - 1)\hbar^2,$$

where $\Delta E_m = E_{max} - E_{min}$,
$\Delta t_{det} = \sqrt{\langle (n\tau)^2 \rangle - \langle n\tau \rangle^2} = \tau \sqrt{Var(n)}$

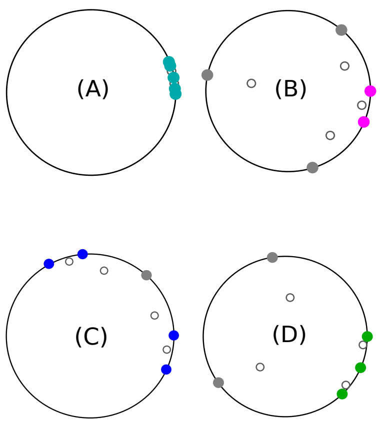
In the mathematical limit $\langle n \rangle = w = 1$, the particle is detected at the first attempt for any sampling time τ.

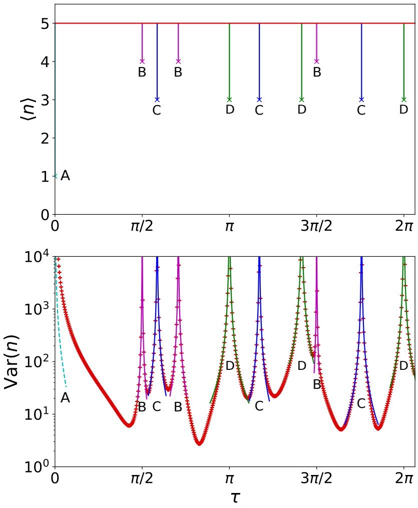
$$\Delta t_{det} = 0$$

The presence of the factor $\langle n \rangle - 1$ is physically reasonable.

Example

Eight-Site Ring





Interacting-Boson Model

$$H = -\frac{J}{2}(\hat{a}_l^\dagger \hat{a}_r + \hat{a}_r^\dagger \hat{a}_l) + U(\hat{n}_l^2 + \hat{n}_r^2), \quad \hat{n}_{l,r} = \hat{a}_{l,r}^\dagger \hat{a}_{l,r}$$

$$E_0 = 3U - \sqrt{U^2 + J^2}, E_1 = 4U, E_2 = 3U + \sqrt{U^2 + J^2}.$$

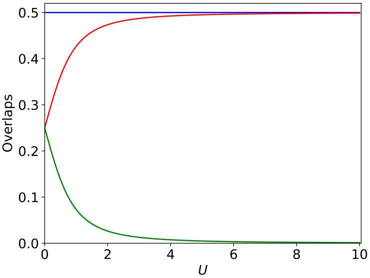


Figure: $|\psi_{in}\rangle = |2, 0\rangle$, and J is set as 1. The green curve represents p_0, and the blue/red is p_1/p_2. U is large, the ground state is almost $|1, 1\rangle$.

Another Example

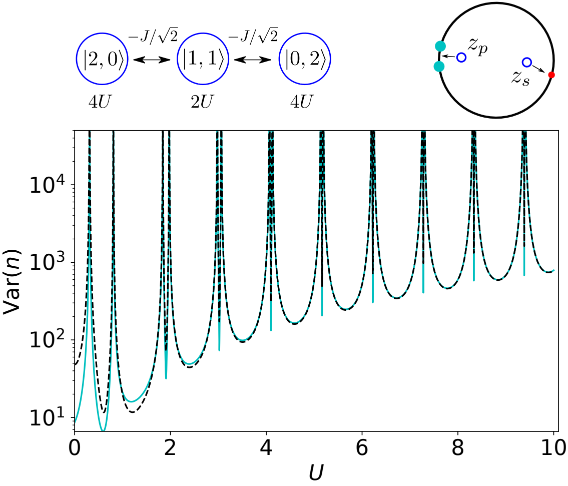


Figure: J = 1, $\tau = 3$

Summary

- Diverging Var(n) accompany isolated jumps of $\langle n \rangle$
- We quantify the magnitude of large fluctuations of first detected return time based on four scenarios: single-weak charge, two-charge merging, three-charge merging, Zeno regime
- Topology-dependent time-energy uncertainty relation

Thank You!

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