Structural reduction of a discrete-time quantum walk

Václav Potoček
Faculty of Nuclear Sciences and Physical Engineering,
Czech Technical University in Prague, Czech Republic

We discuss an intriguing result that summation of probability amplitudes of a discrete-time quantum walk over partitions of the walking graph consistent with the step operator results in a unitary evolution on the reduced graph which is also a quantum walk. Since the reduced walking graph is not necessarily simpler, in an intuitive sense, than the original, this may bring new insights into the dynamics of some kinds of quantum walks using known results from thoroughly studied cases like Euclidean lattices. This is not to be confused with other methods of simplifying the problem utilizing its symmetries or proving decomposition into a tensor product of independent evolutions, although comparisons are made and the approaches can potentially be combined.

We use abstract treatment of the walking space and walker displacements in aim for a generality of the presented statements. Using this approach we also identify some pathological cases in which the reduction mapping breaks down. Nevertheless, we illustrate the general theory on straightforward examples using Euclidean lattices.

For walks on lattices, the operation results in quantum walks with hyper-dimensional coin spaces. Such walks can, on the reverse, be viewed as reductions of walks on inaccessible, larger spaces, and their properties can be inferred from the parental walk. We show that this is is the case for a lazy quantum walk, a walk with large coherent jumps and a walk on a circle with a twisted boundary condition. Moreover, this manifestly irreversible operation can, in some cases and with a minor adjustment, be undone, and a quantum walk can be reconstructed from a set of its reductions.

Finally, we discuss the relation of this theory to the time-multiplexing optical implementations of a quantum walks.